

Modélisation physique des erreurs de modèles pour les écoulements fluides géophysiques

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Pierre Dérian,
Etienne Mémin,
Bertrand Chapron

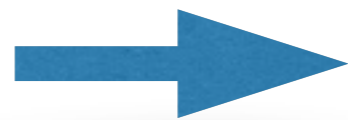
Motivations

- Identifier rigoureusement l'effet de la dynamique sous-maille
- Injecter une dynamique petite échelle plausible
- Etudier les bifurcations et les attracteurs



Projection climatique

- Quantifier les erreurs de modèle



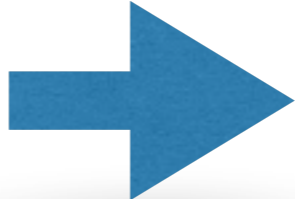
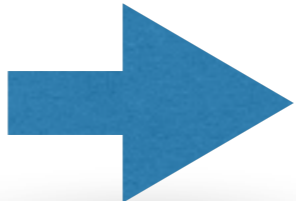


Prévision d'ensemble et assimilation de données

Plan

- Dynamique randomisée
- SQG sous incertitude modérée (SQG MU)
- Lorenz sous incertitude de position

Dynamique randomisée

Introduction d'aléas

- Conditions initiales aléatoires  Sous-dispersif +
Nécessite un large
ensemble
- Forçage Gaussien arbitraire  Rajoute de l'énergie +
Mauvaise phase
- Moyennage,
homogénéisation  Problèmes d'hypothèses et
de conservation de l'énergie
- Ajout d'une vitesse
décorrélée en temps  $\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$

Advection d'un traceur Θ

$$\frac{D\Theta}{Dt} = 0$$

$$v = w + \sigma \dot{B}$$

Advection d'un traceur Θ

$$v = w + \sigma \dot{B}$$

Advection d'un traceur Θ

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{B}$$

Advection d'un traceur Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

$$v = w + \sigma \dot{B}$$

Advection d'un traceur Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

$$v = w + \sigma \dot{B}$$

Advection d'un traceur Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)_{\text{Diffusion}}$$

Correction
du drift

$$v = w + \sigma \dot{B}$$

Advection d'un traceur Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Correction du drift}} + \underbrace{\sigma \dot{B} \cdot \nabla \Theta}_{\text{Forçage aléatoire multiplicatif}} = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection

Diffusion

$$v = w + \sigma \dot{B}$$

Advection d'un traceur Θ

Advection

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

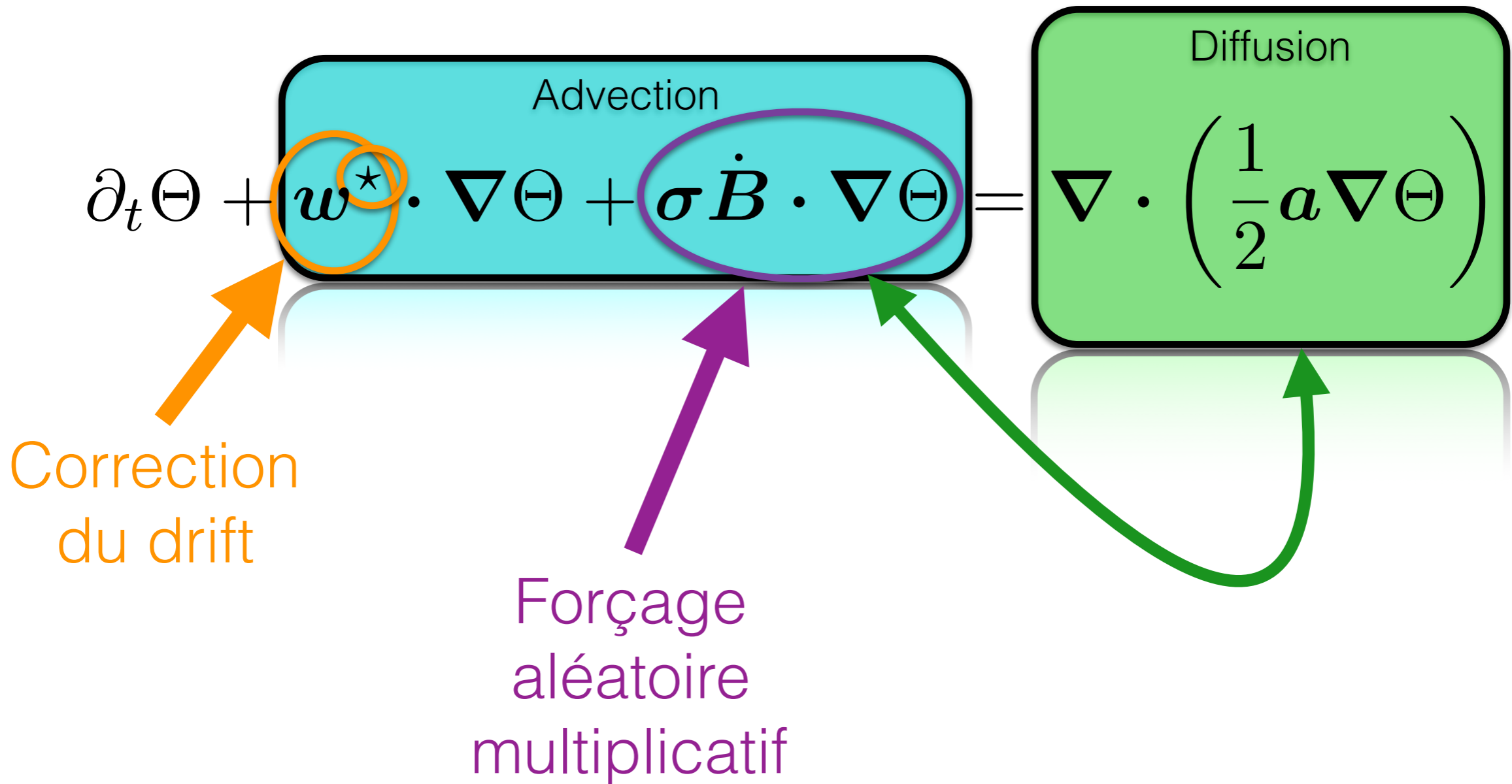
Diffusion

Correction du drift

Forçage aléatoire multiplicatif

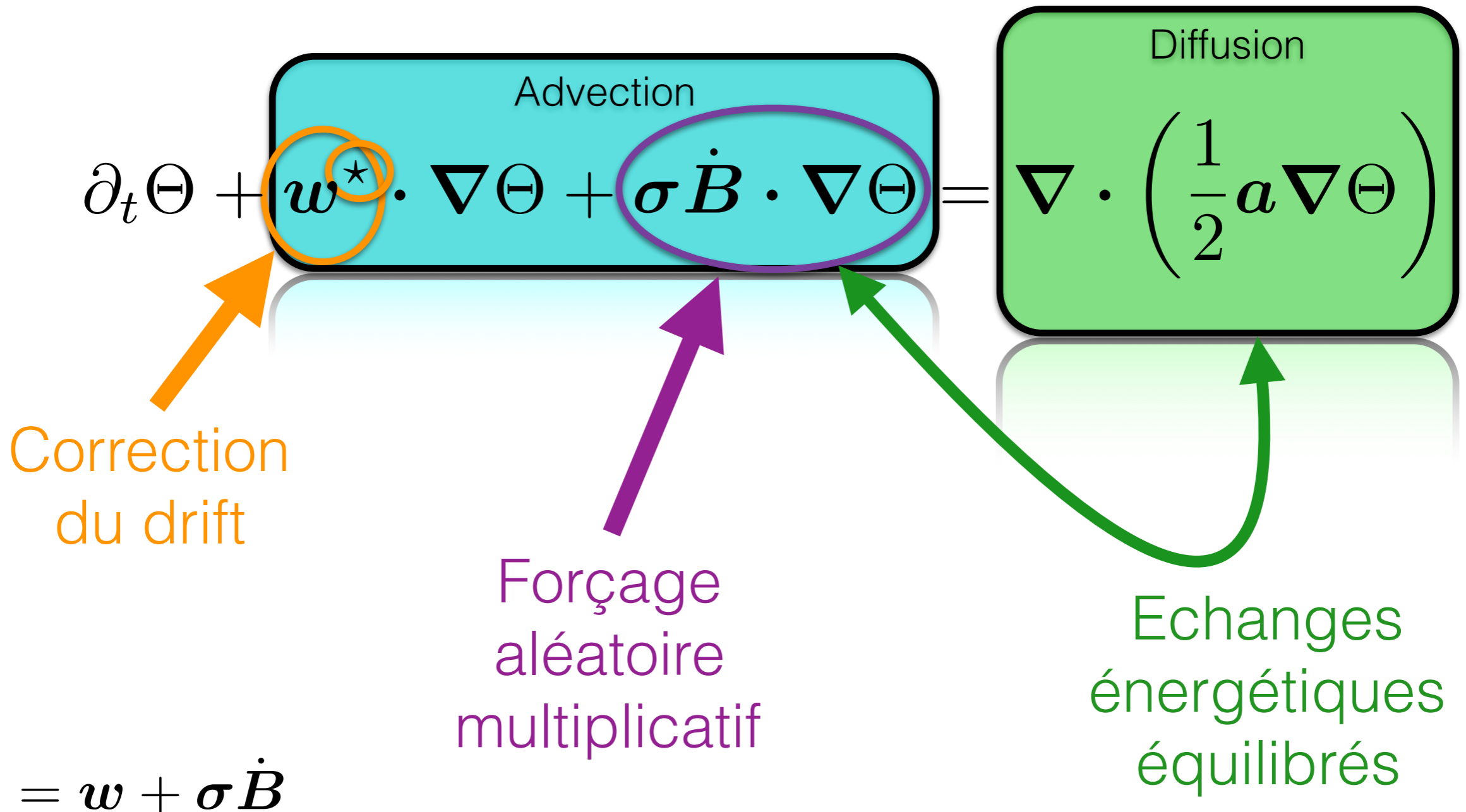
$$v = w + \sigma \dot{B}$$

Advection d'un traceur Θ

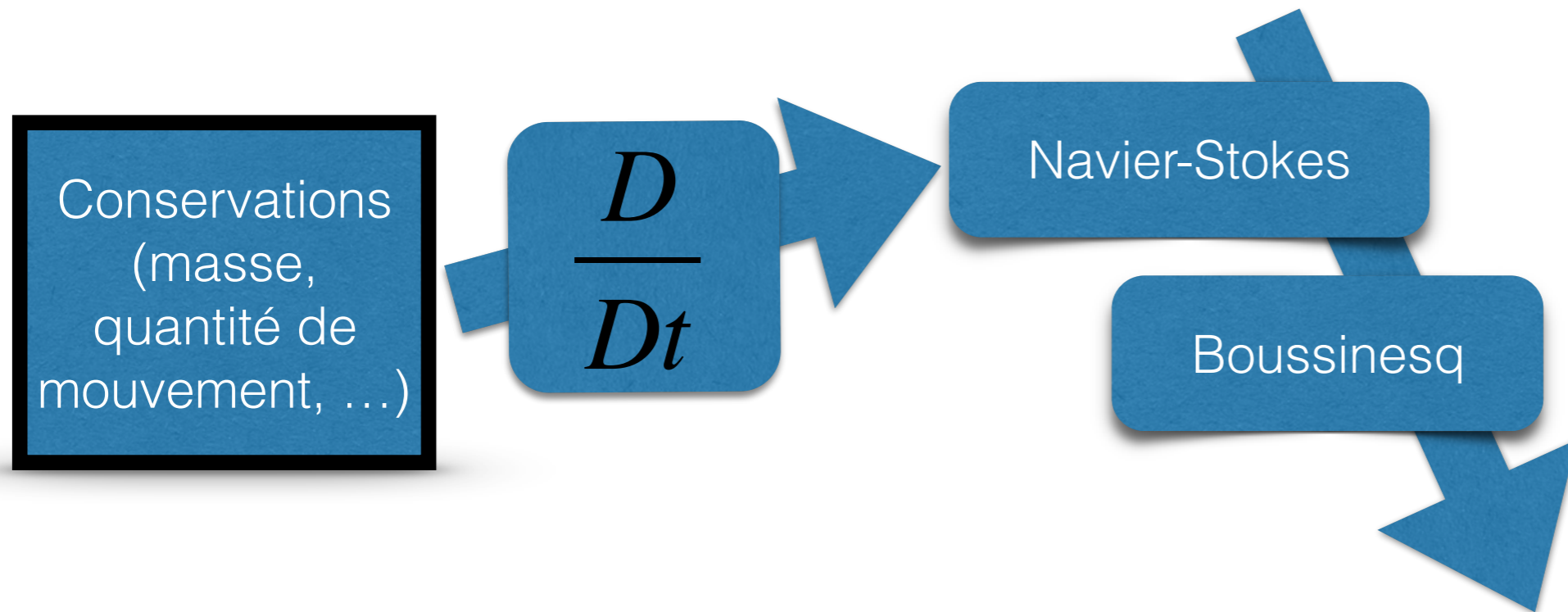


$$v = w + \sigma \dot{B}$$

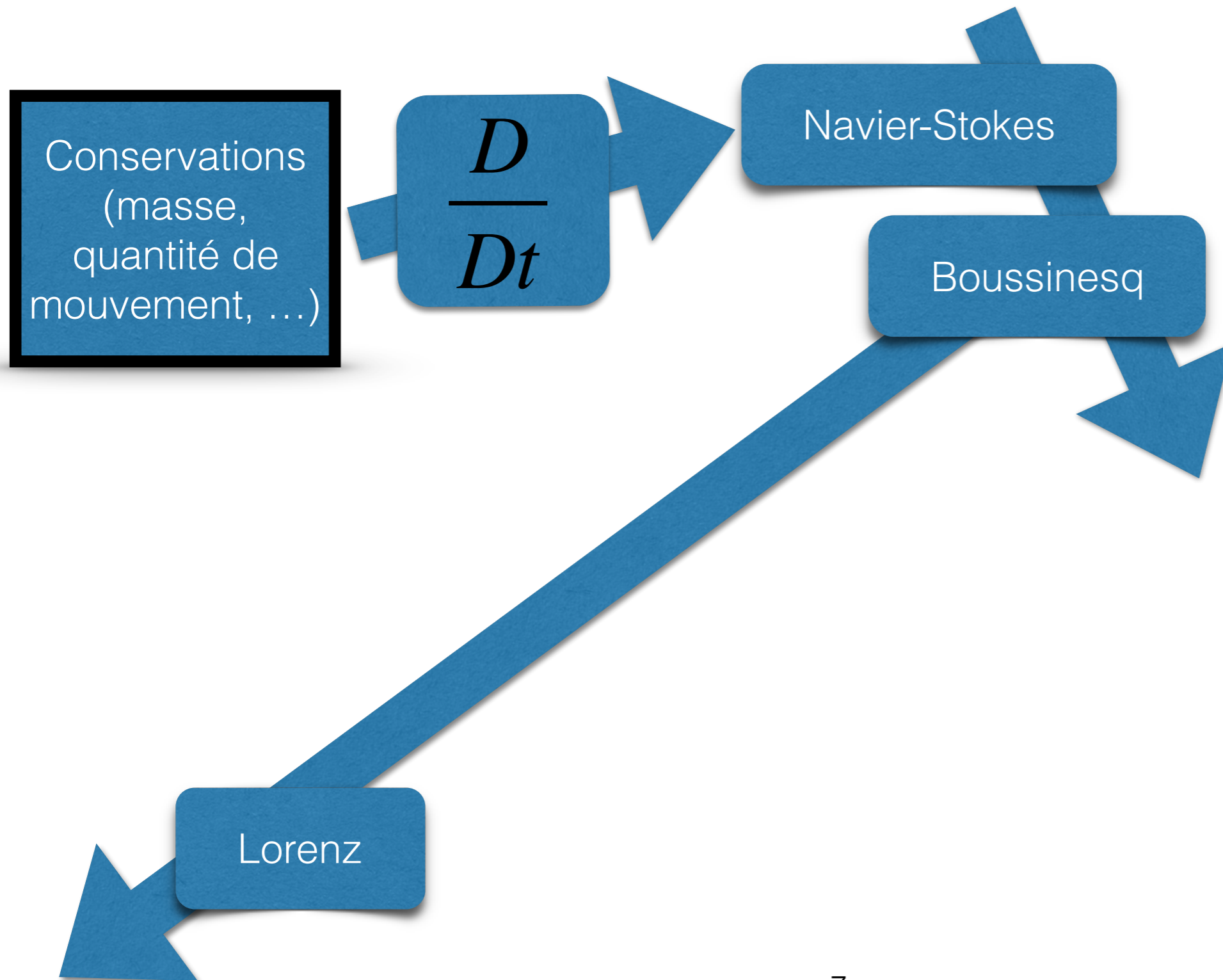
Advection d'un traceur Θ



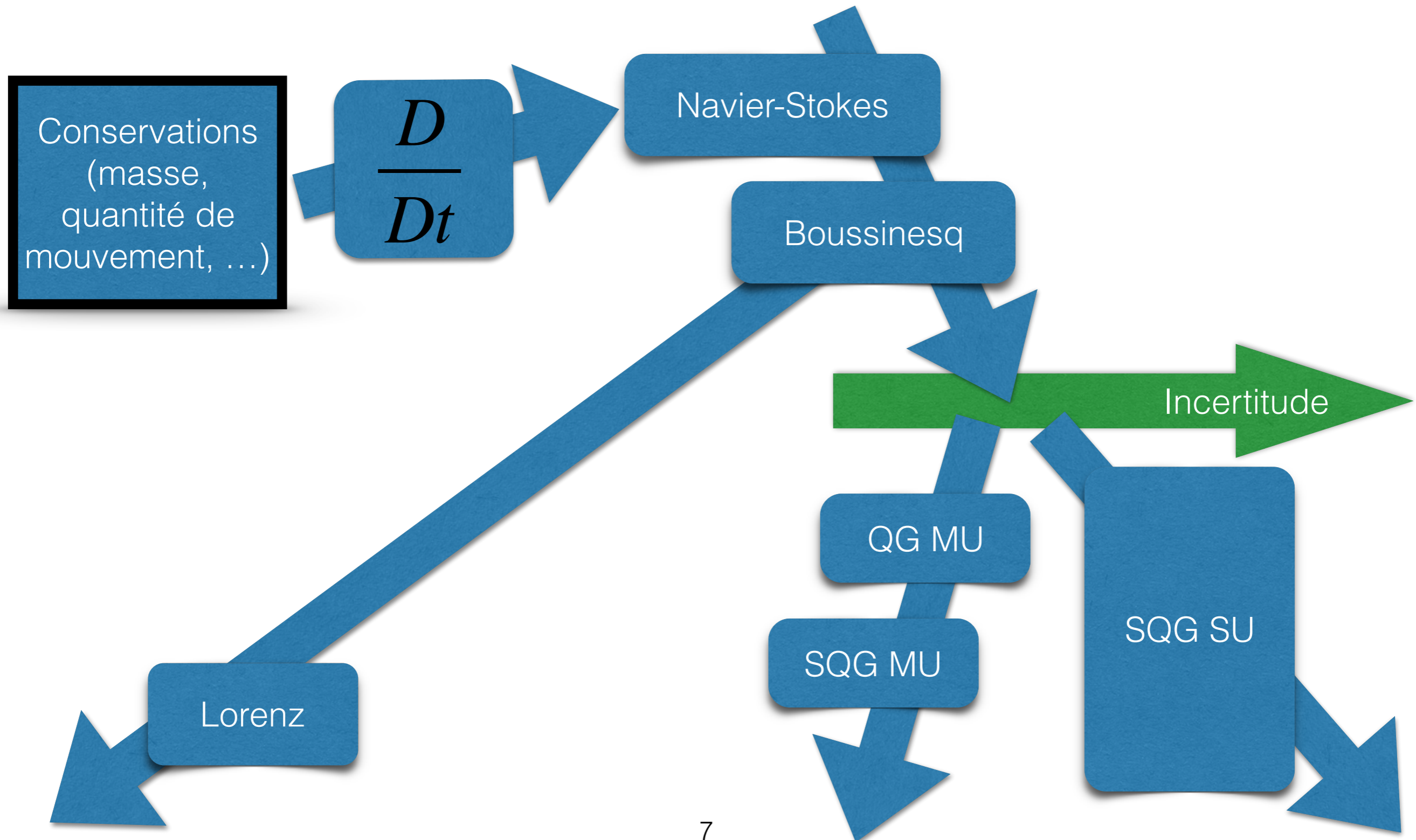
Modèles aléatoires dérivés



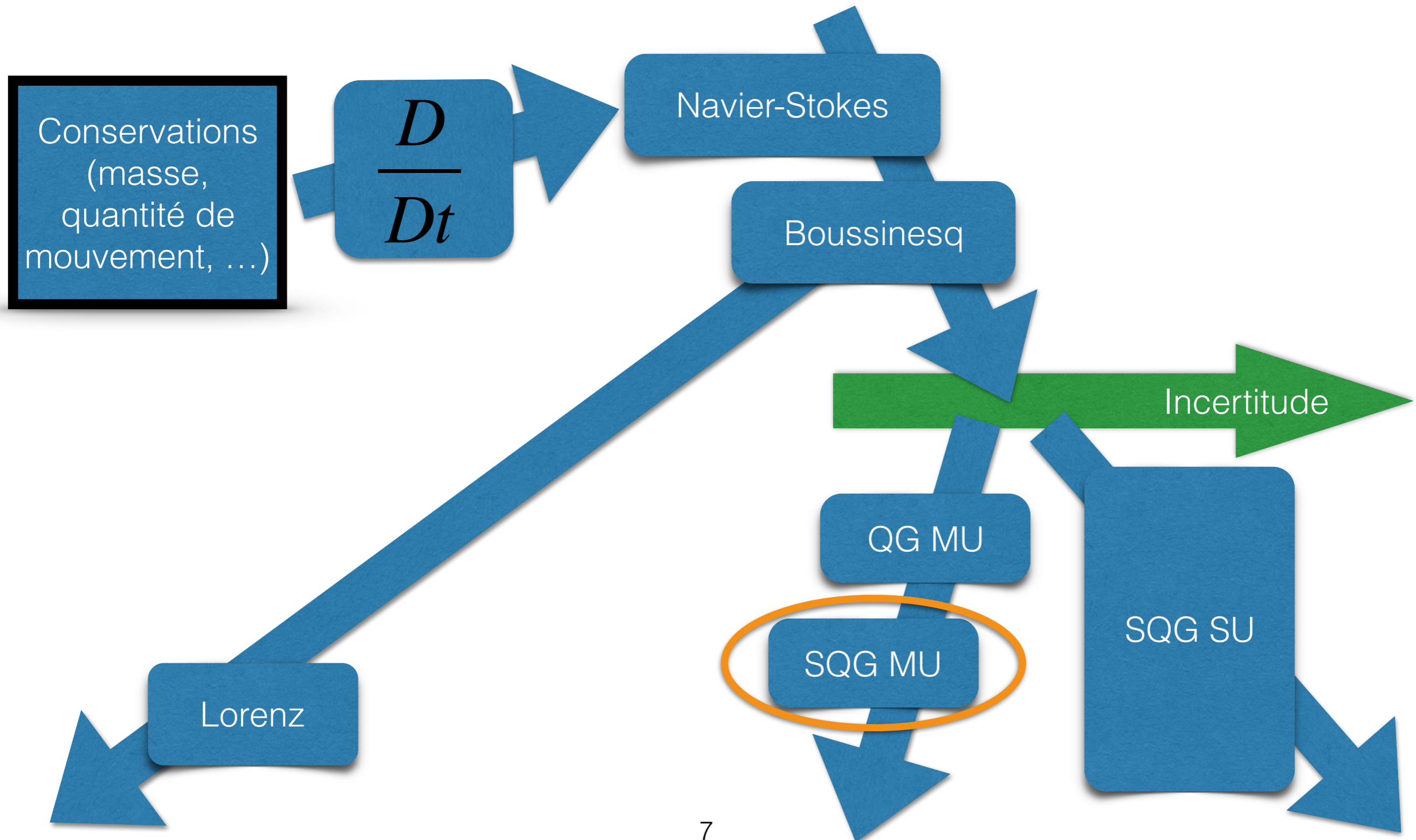
Modèles aléatoires dérivés



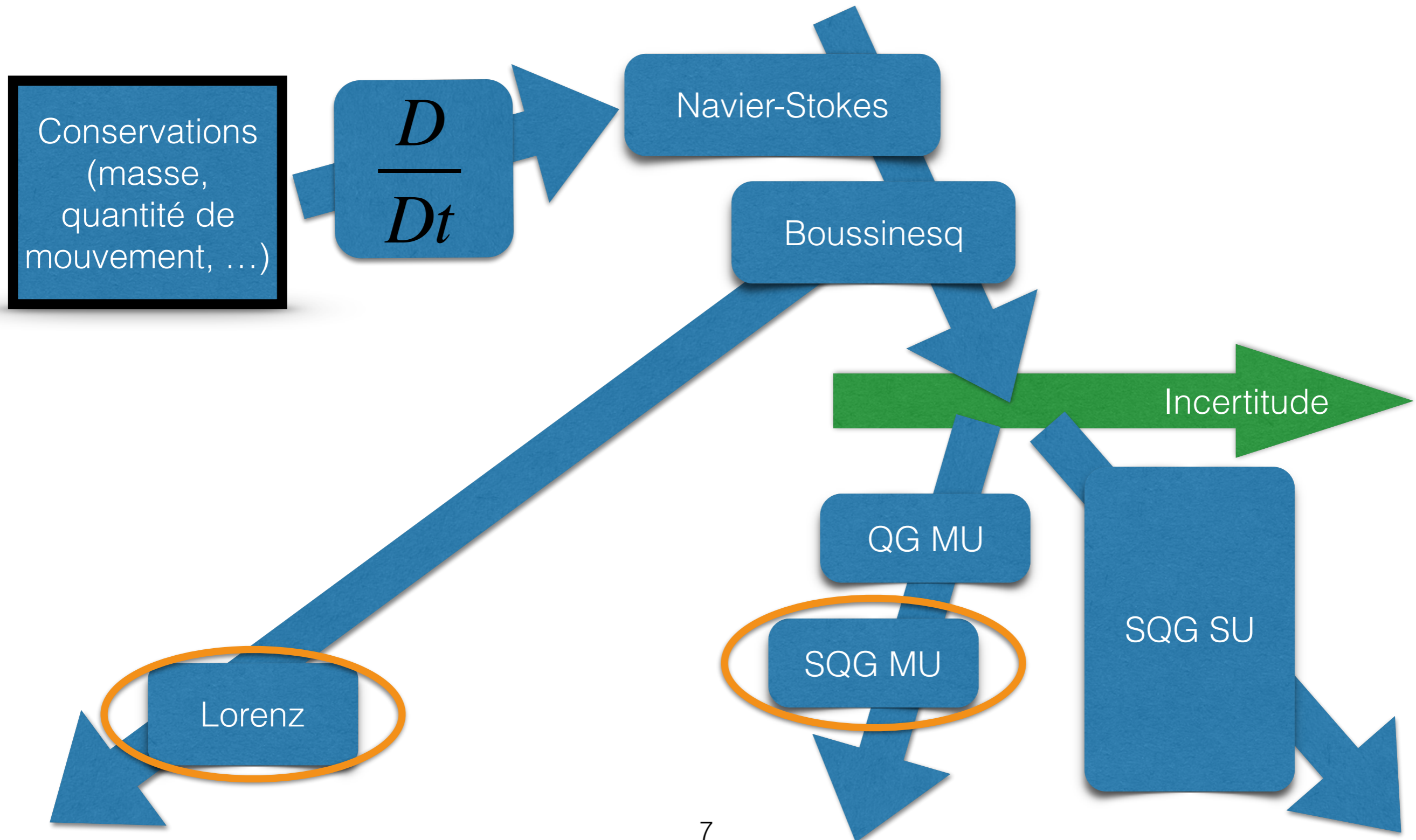
Modèles aléatoires dérivés



Modèles aléatoires dérivés



Modèles aléatoires dérivés



SQG sous incertitude modérée

SQG MU

Code disponible en ligne

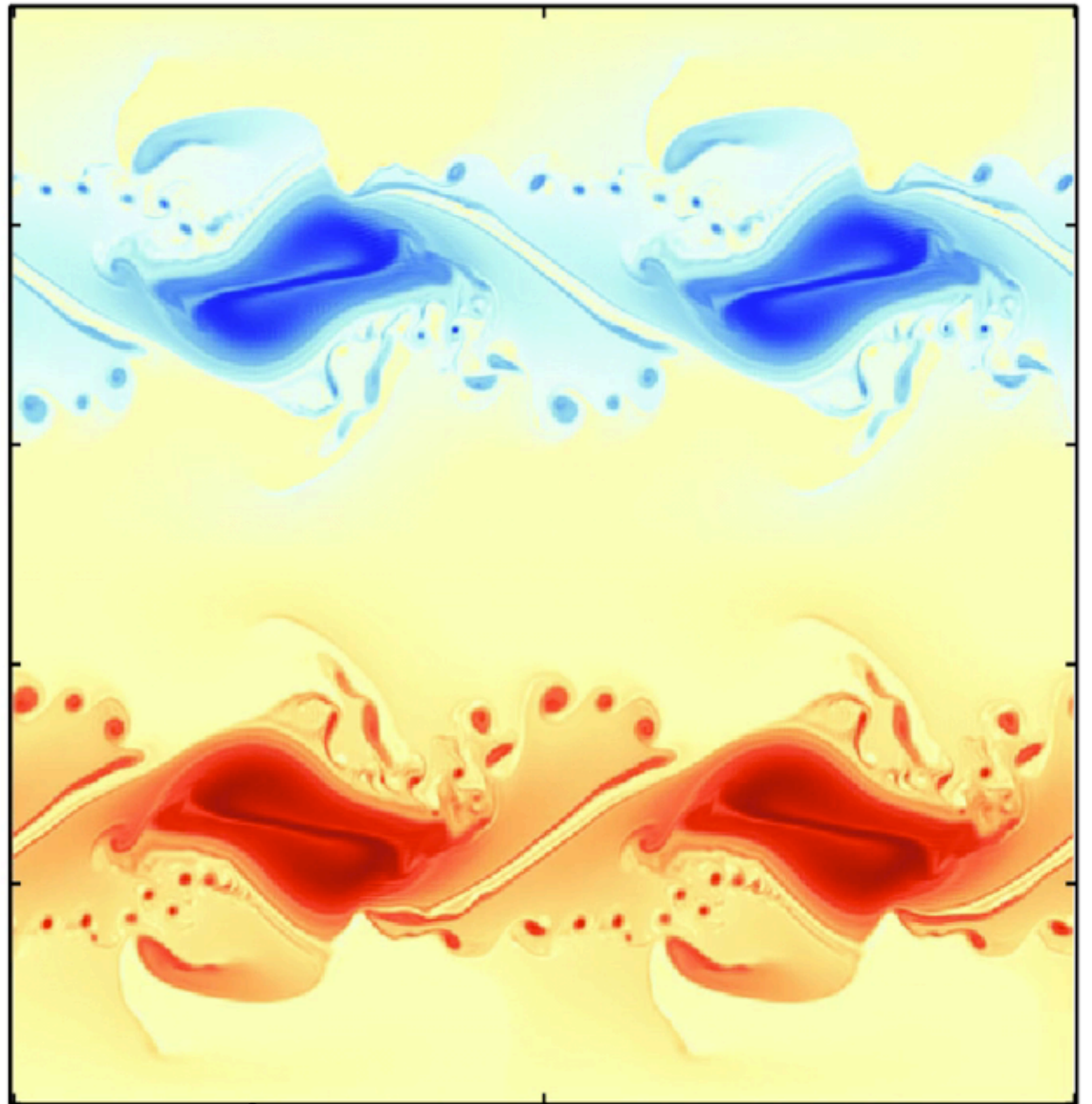
$t = 17$ days

Simulation de
référence:

SQG

déterministe

512 x 512



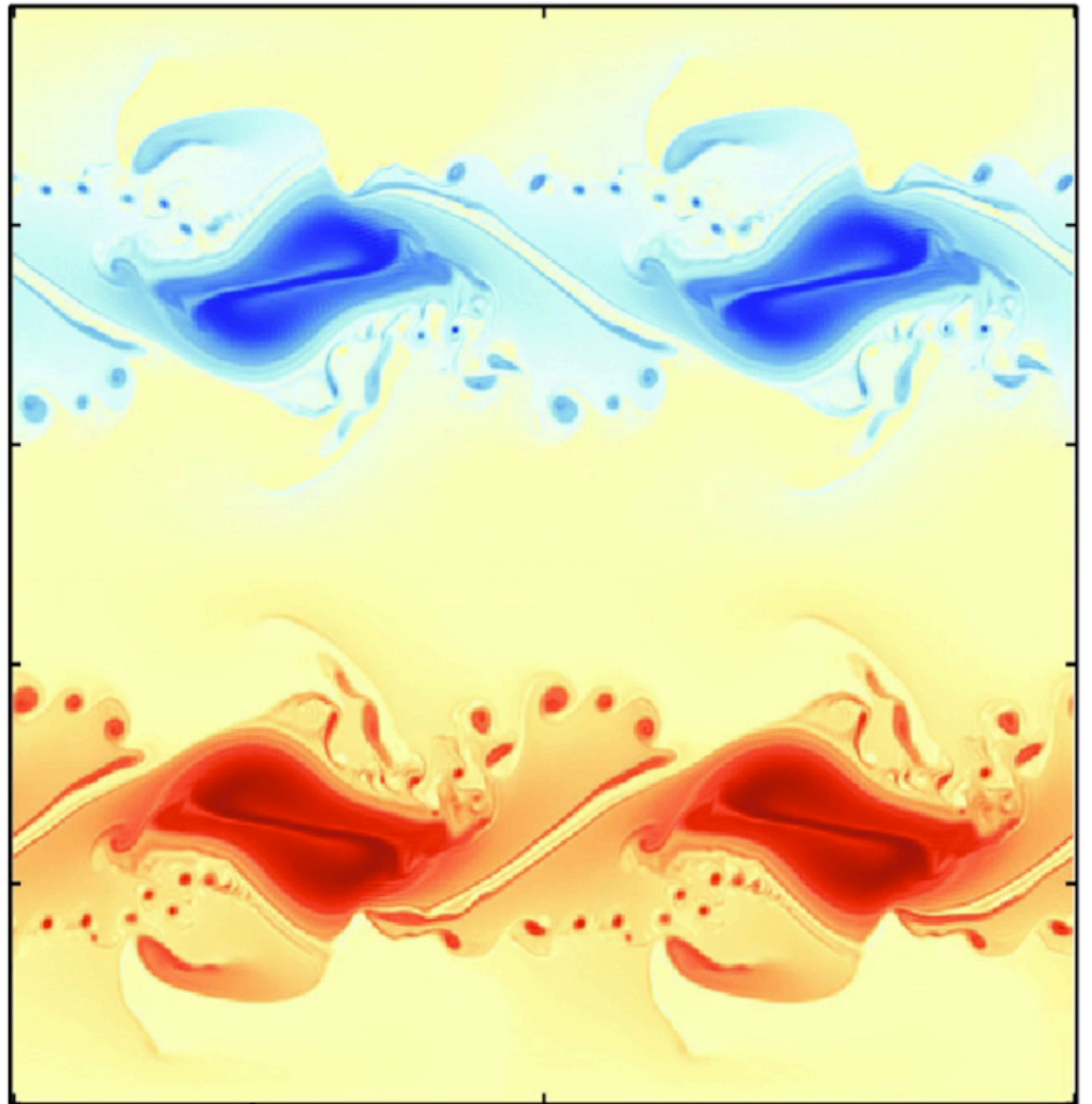
$t = 17$ days

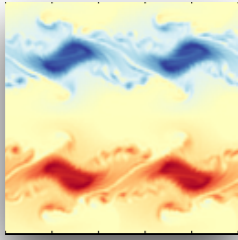
Simulation de
référence:

SQG

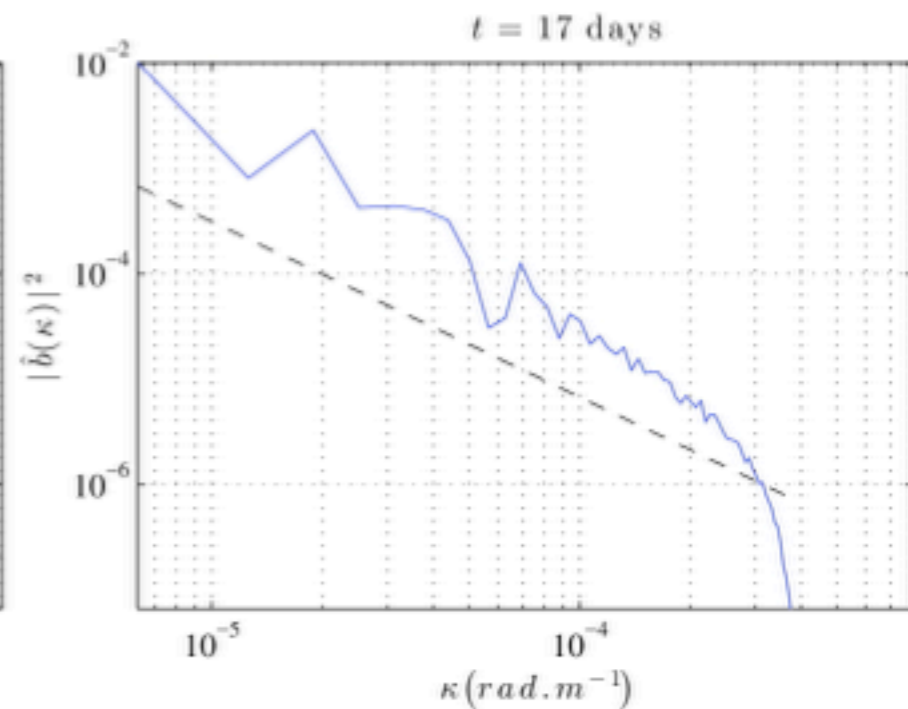
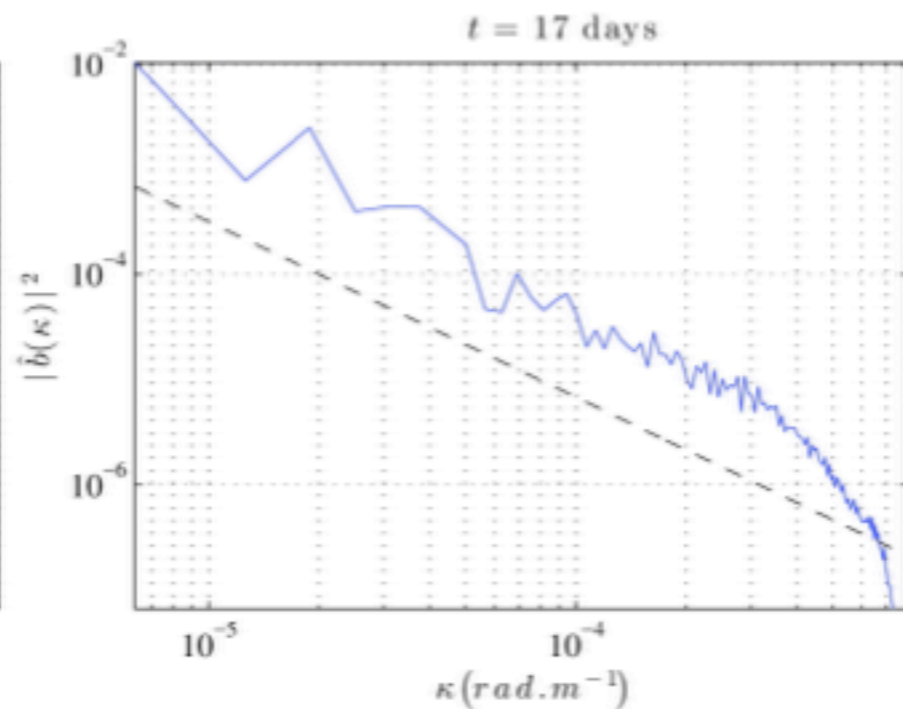
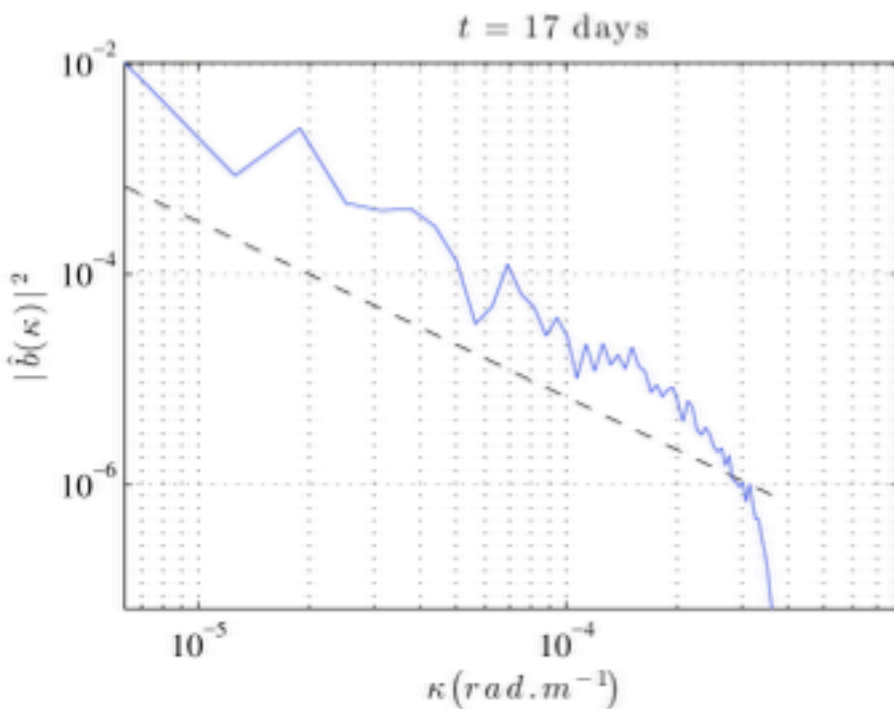
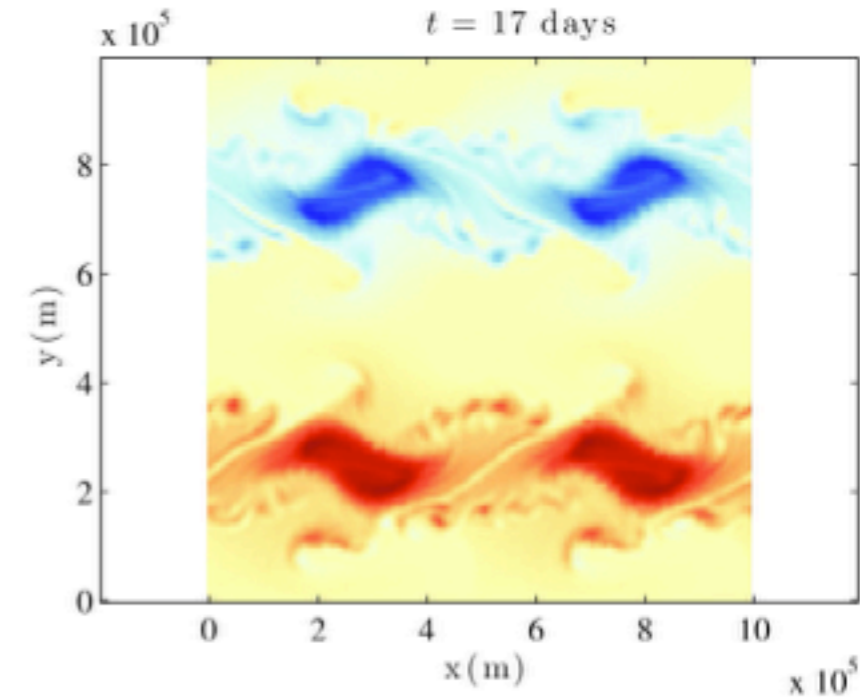
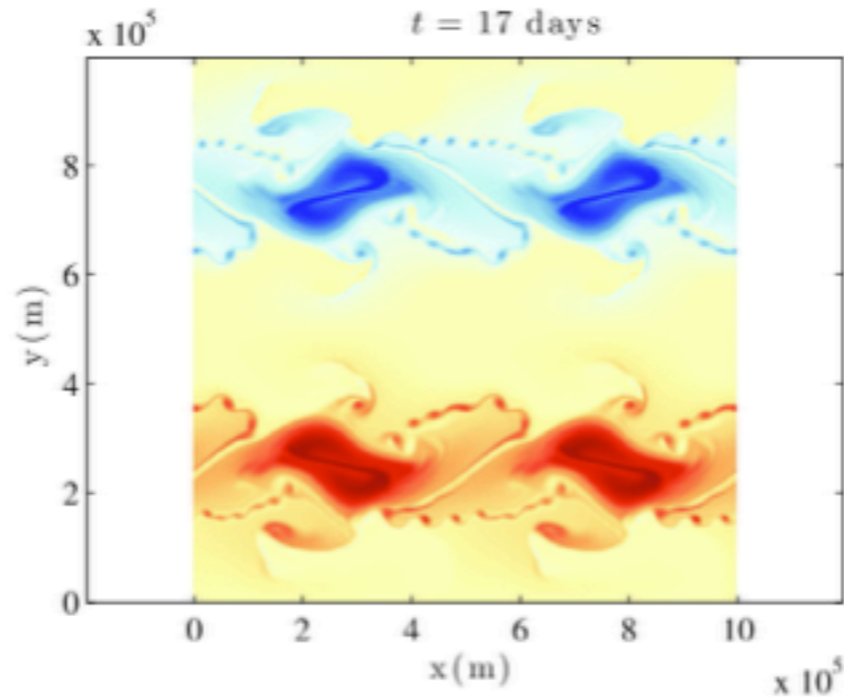
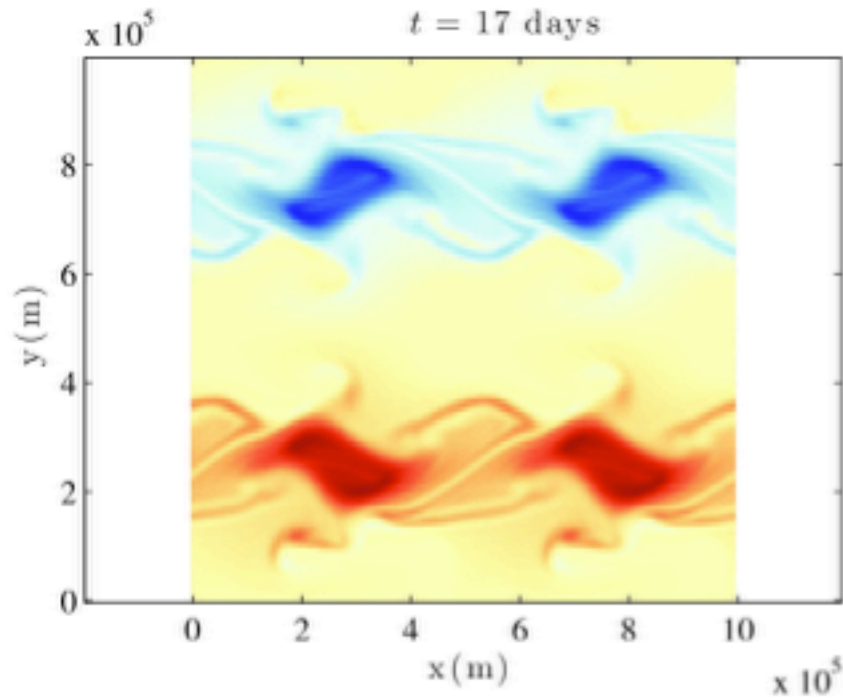
déterministe

512 x 512





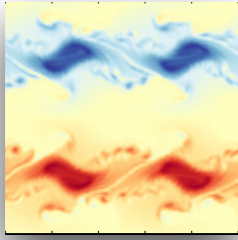
Une réalisation



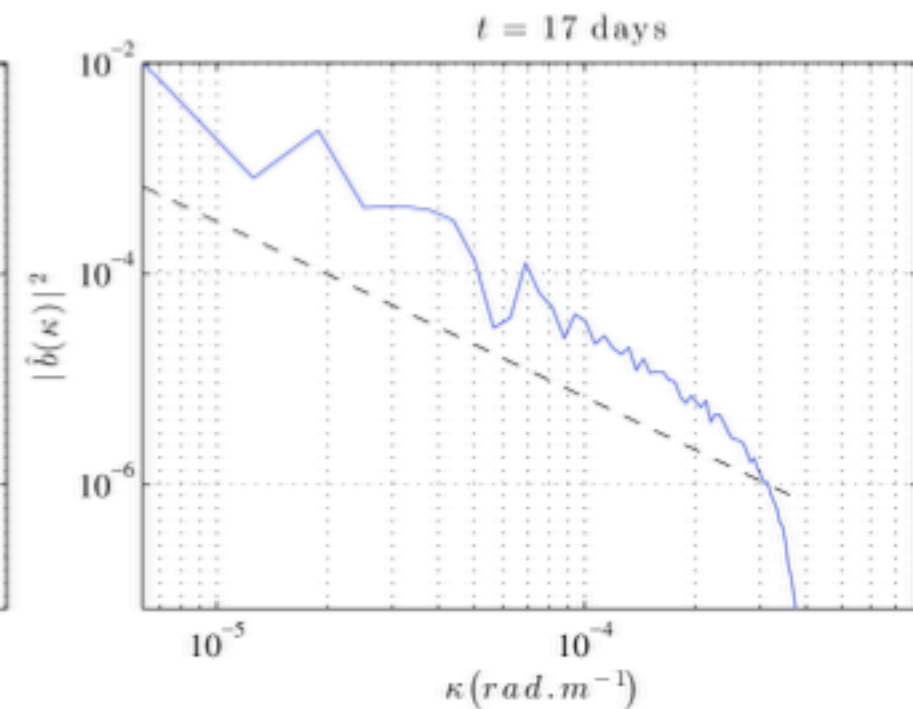
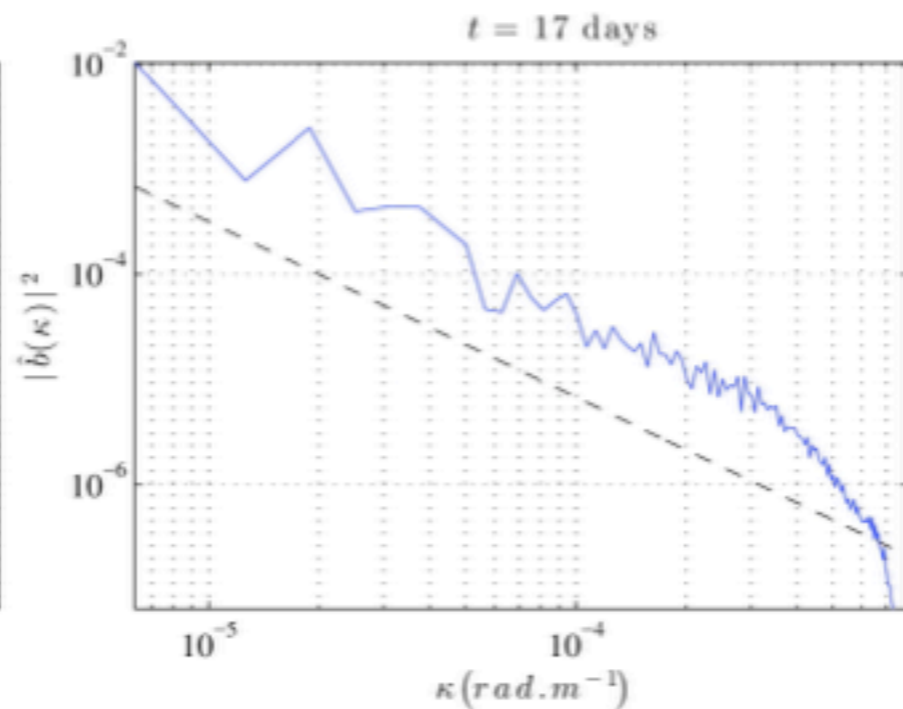
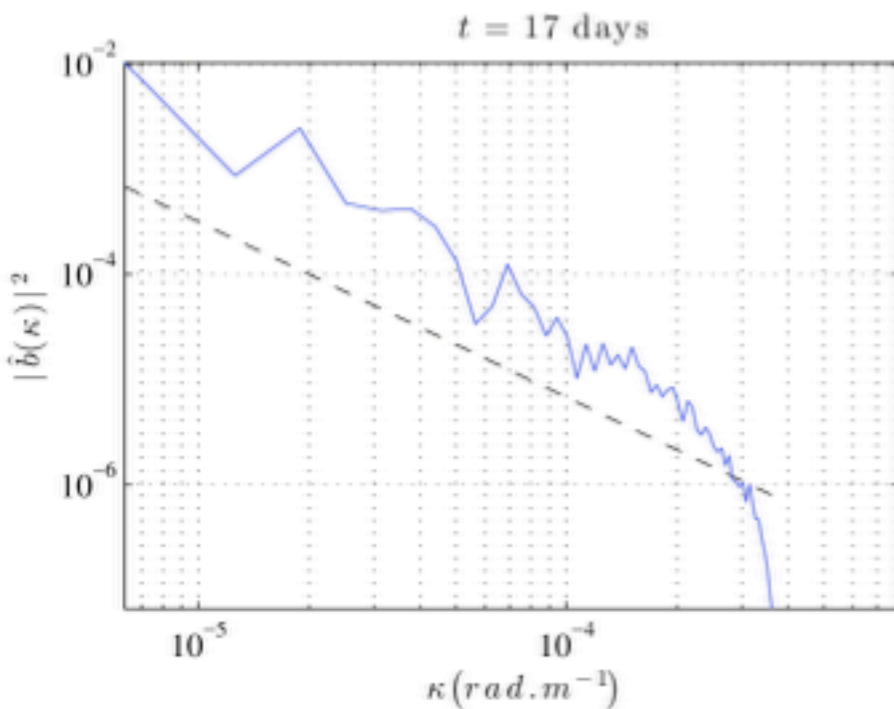
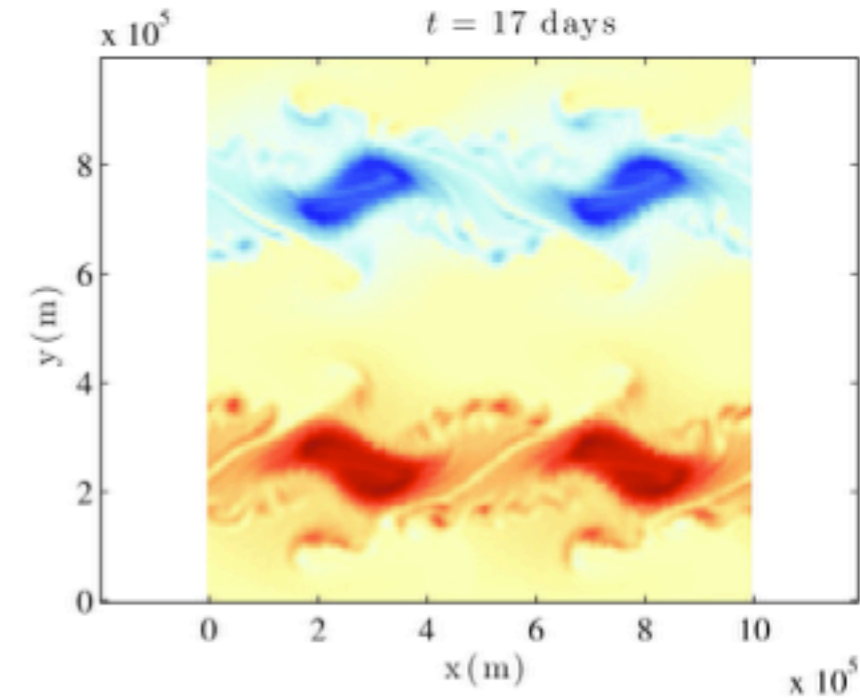
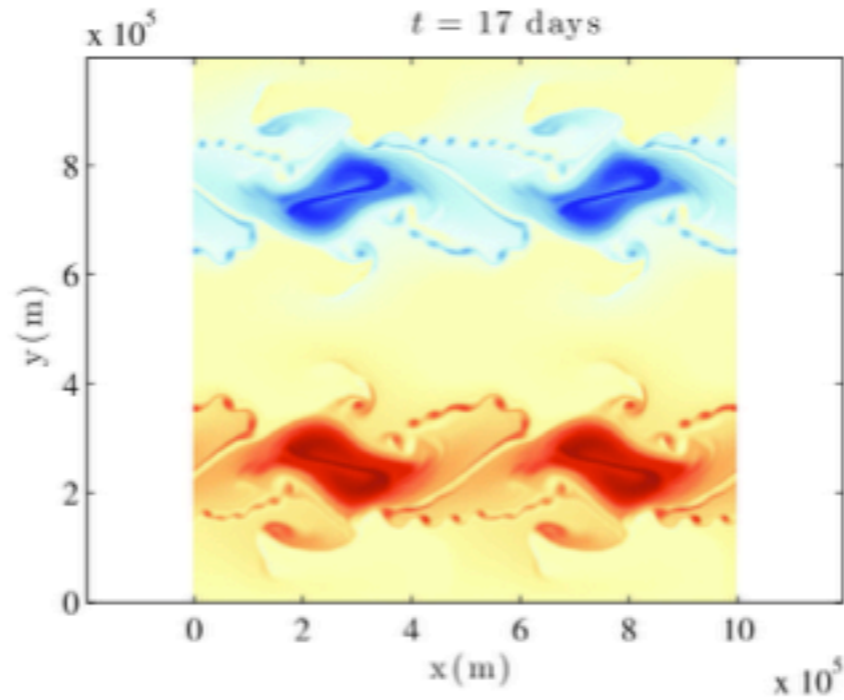
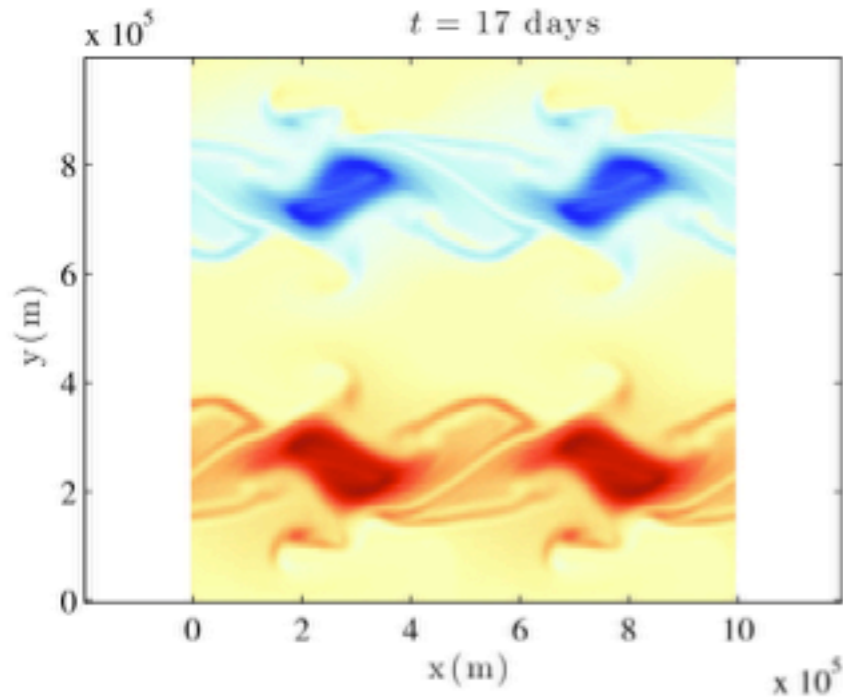
Deterministe 128x128

Deterministe 512x512

Stochastique 128x128



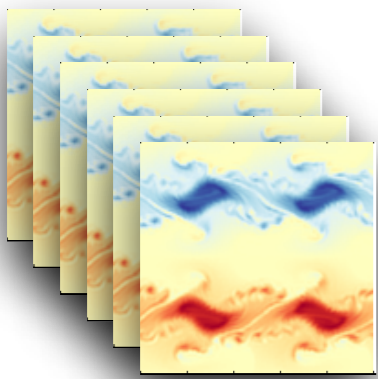
Une réalisation



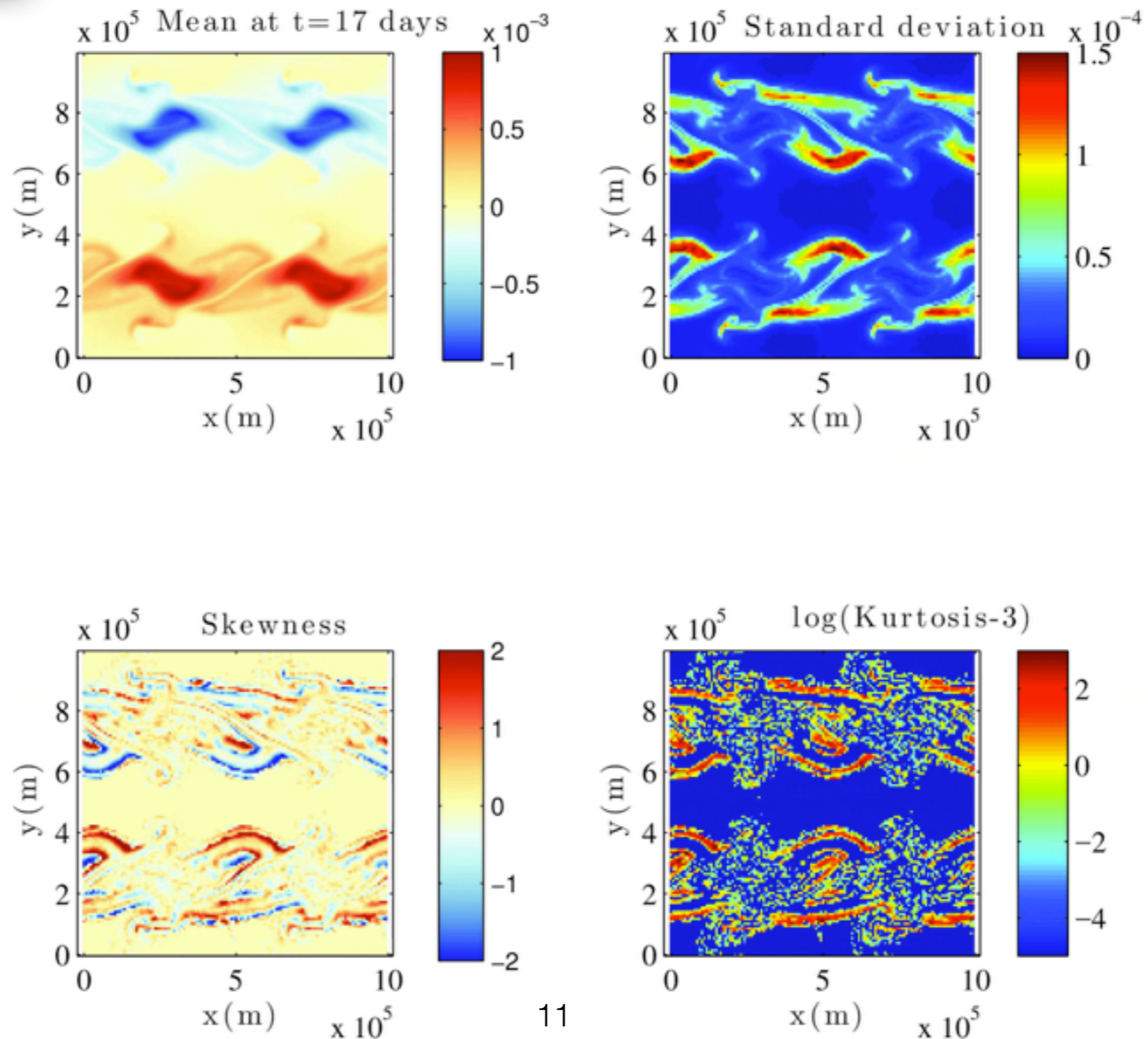
Deterministe 128x128

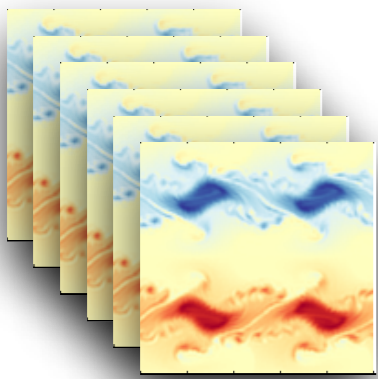
Deterministe 512x512

Stochastique 128x128

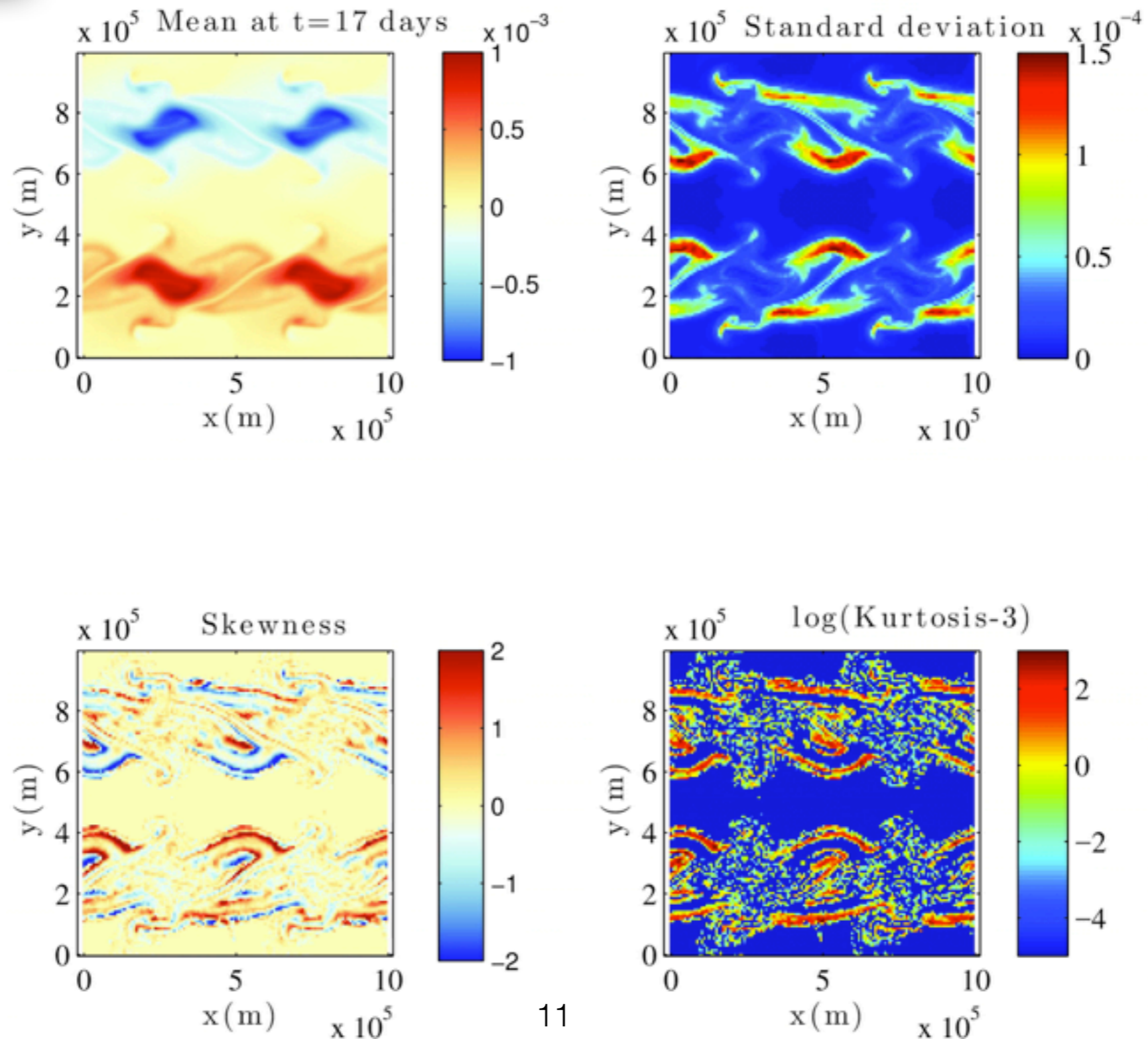


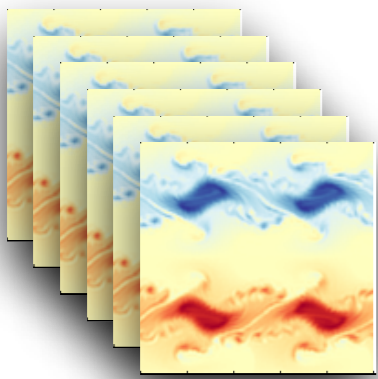
Ensemble



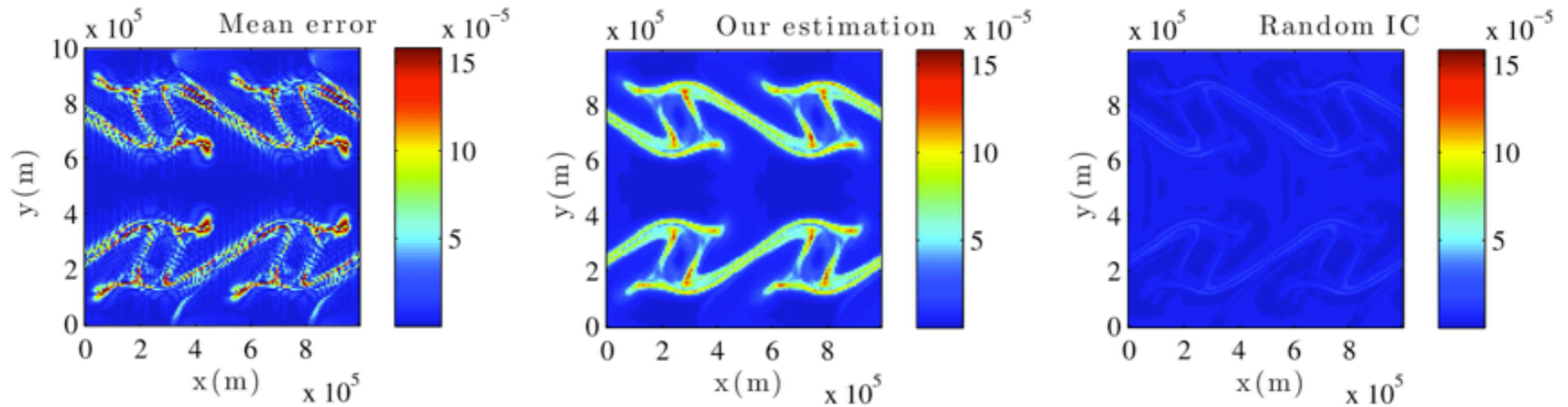


Ensemble

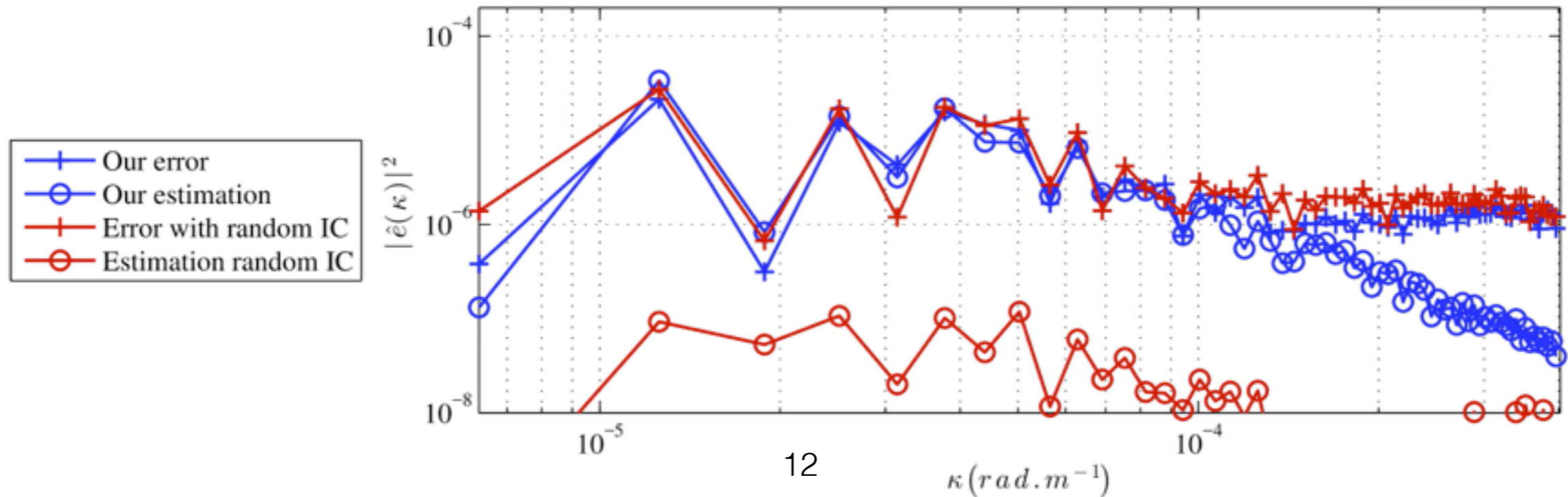


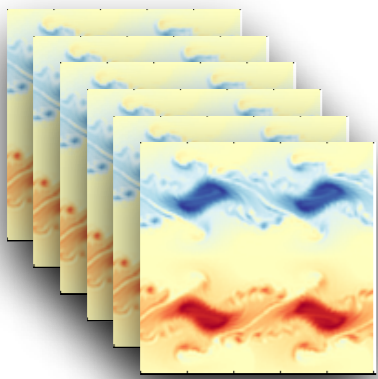


Ensemble

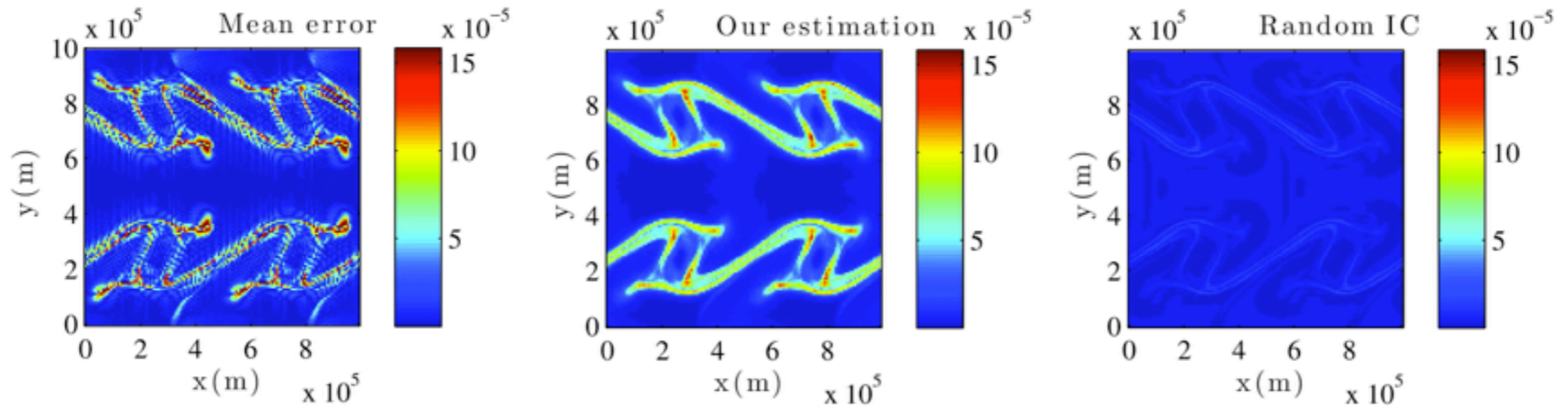


Spectrum of the errors and its estimation at $t=12$ days

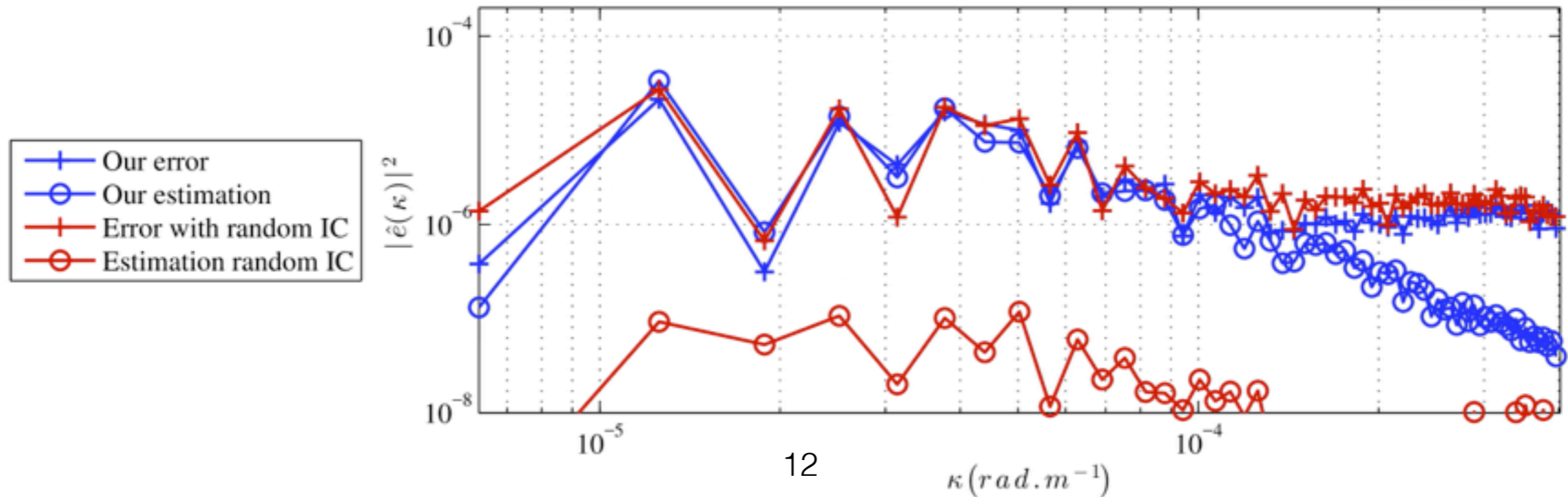




Ensemble

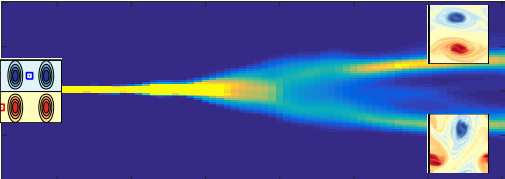


Spectrum of the errors and its estimation at $t=12$ days



Résumé des modèles QG sous incertitude de position

- Petites échelles plus réalistes
- Estimation de la position et de l'amplitude des erreurs
- Évènements extrêmes

- Bifurcations 

- sous incertitude forte:
Description 2D simple de la frontolyse/frontogénèse

Code SQG MU:

lien depuis le site de l'équipe Fluminance - V. Resseguier

Modèle de Lorenz sous incertitude de position

Est-ce que les modèles grande échelle
(diffusifs) sur-représentent les états stables
dans les simulations d'ensemble?

Modèle(s) de Lorenz

$$\frac{dX}{dt} = \text{Pr} (Y - X) - \frac{4}{2\Upsilon} X$$

$$dY = \left[X(\rho - Z) - Y - \frac{4}{2\Upsilon} Y \right] dt + \frac{\rho - Z}{\Upsilon^{1/2}} dB_t$$

$$dZ = \left[XY - bZ - \frac{8}{2\Upsilon} bZ \right] dt + \frac{Y}{\Upsilon^{1/2}} dB_t$$

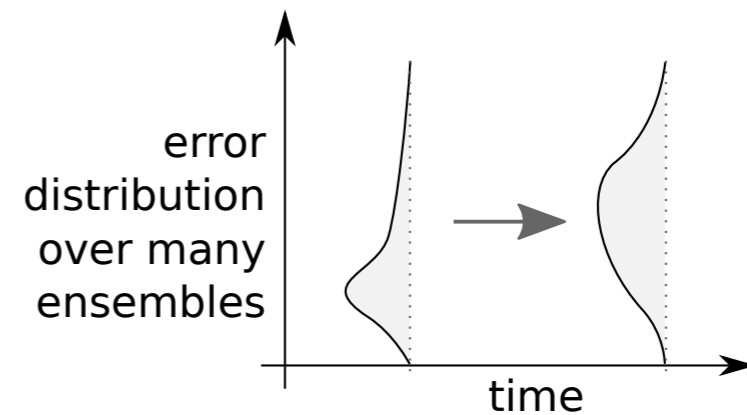
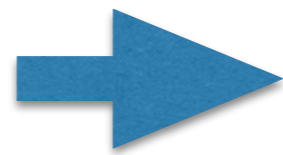
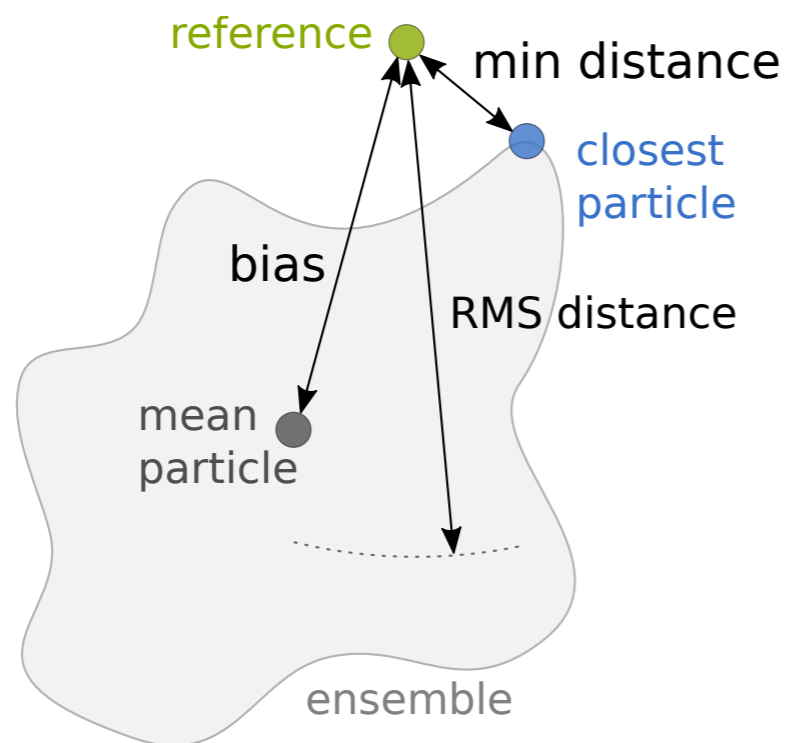
- modèle **déterministe** classique ~ DNS, précis mais inaccessible en pratique
- modèle (déterministe) **diffusif** ~ LES
- **stochastique** : modèle sous incertitude de position

➔ comportement des ensembles?



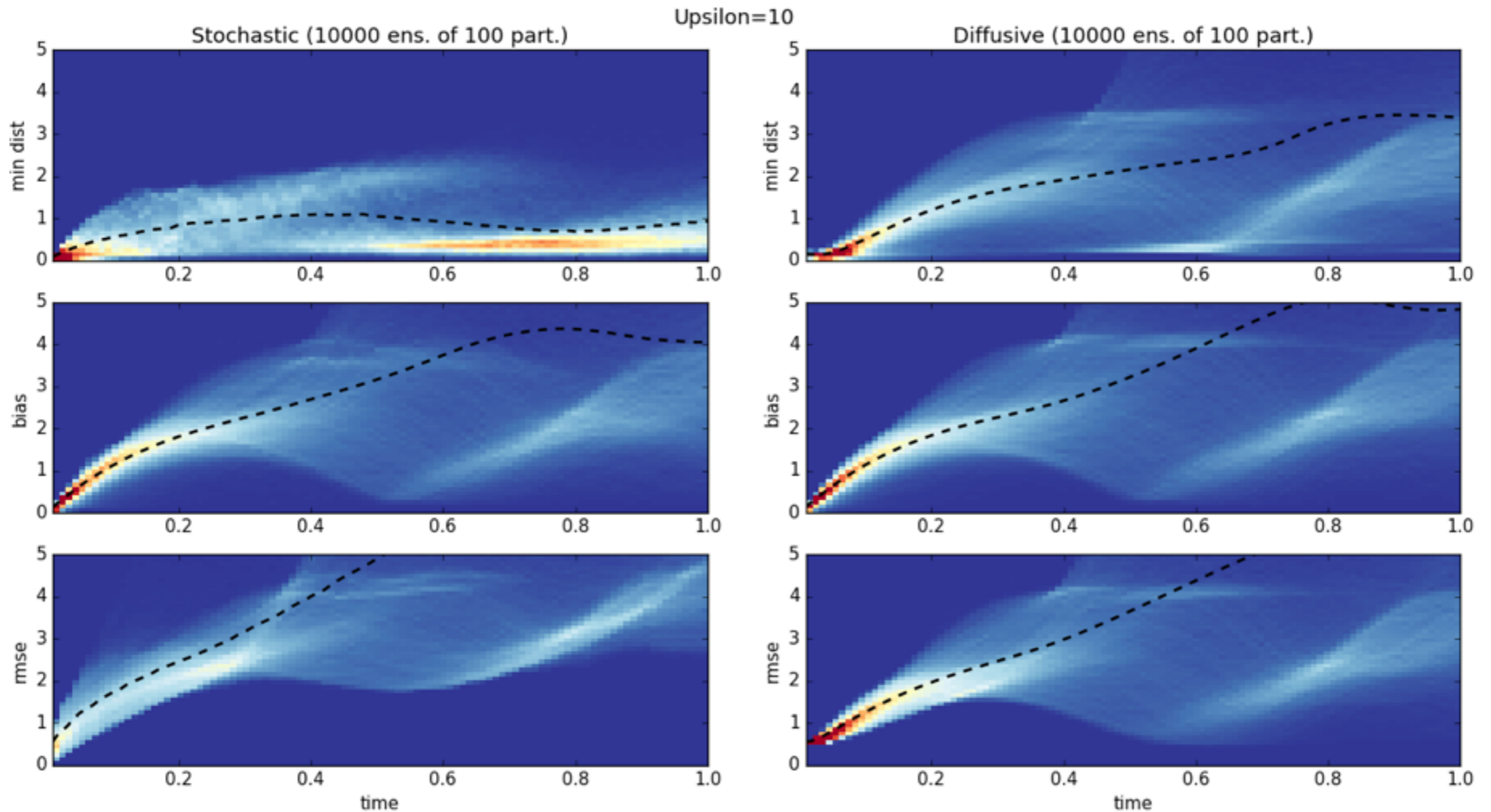
Comportement à temps court

Comparaison ensemble \leftrightarrow référence
3 métriques: distance minimum, biais, RMSE





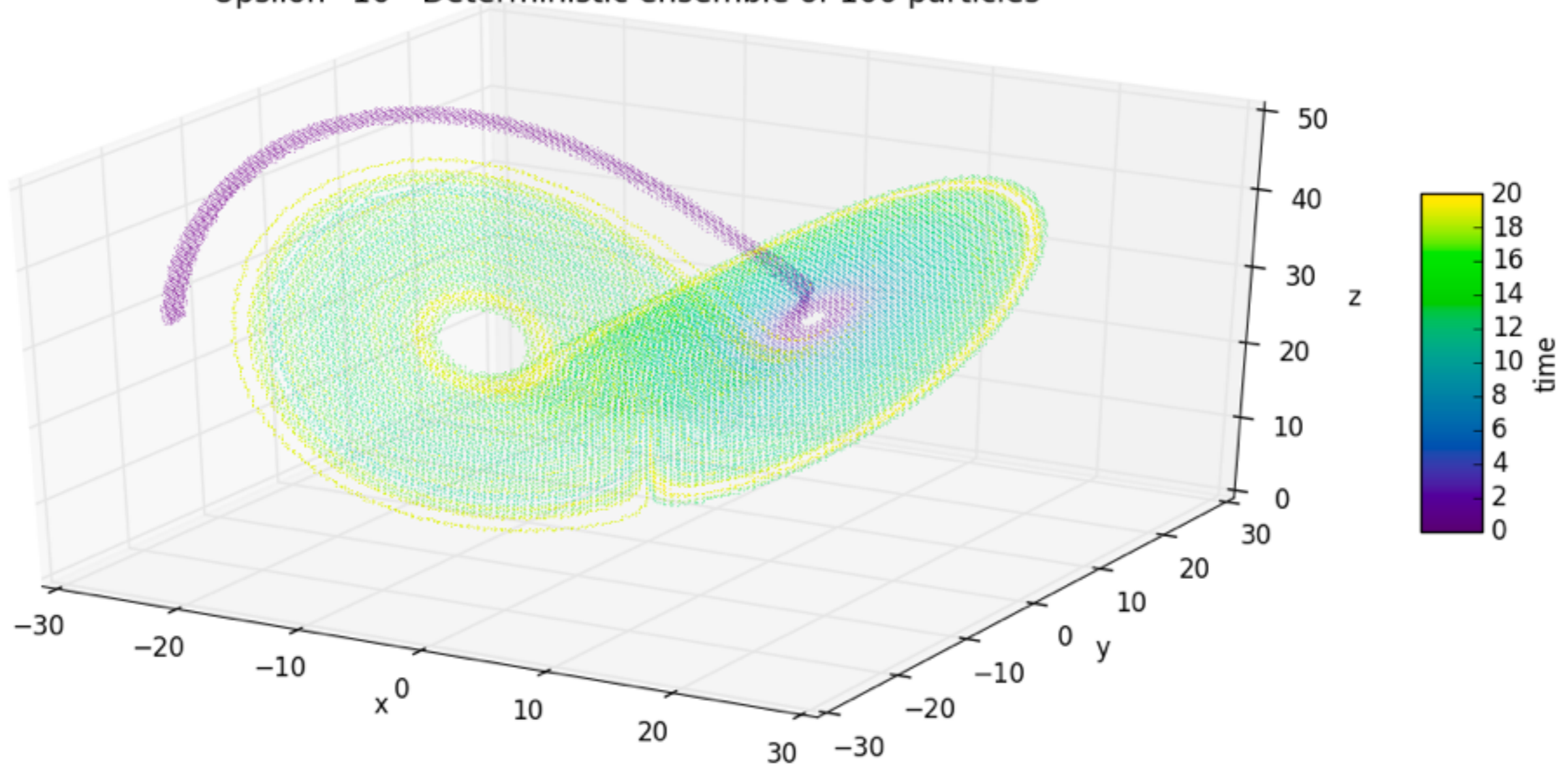
Comportement à temps court





Comportement à temps long

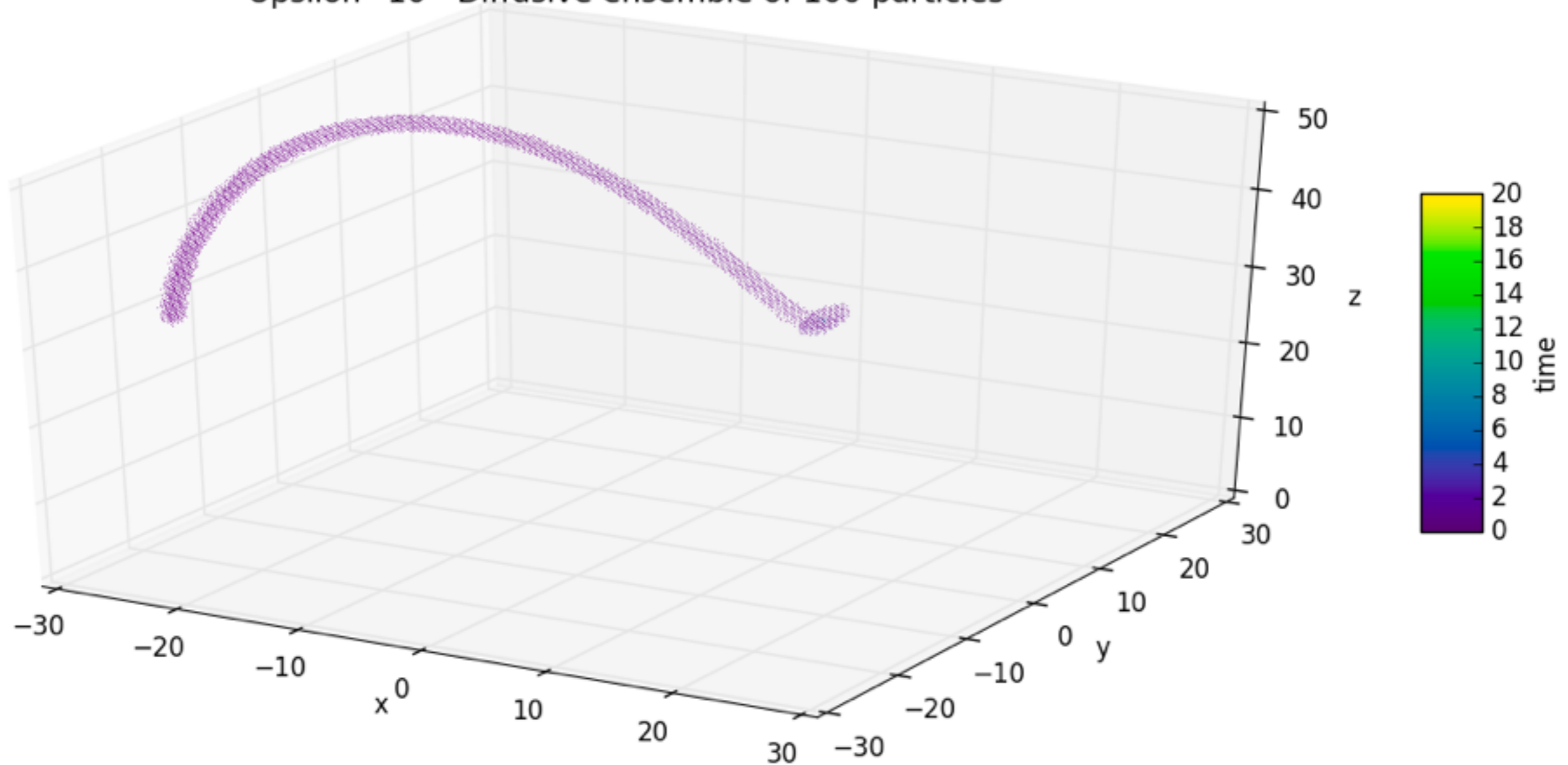
Upsilon=10 - Deterministic ensemble of 100 particles





Comportement à temps long

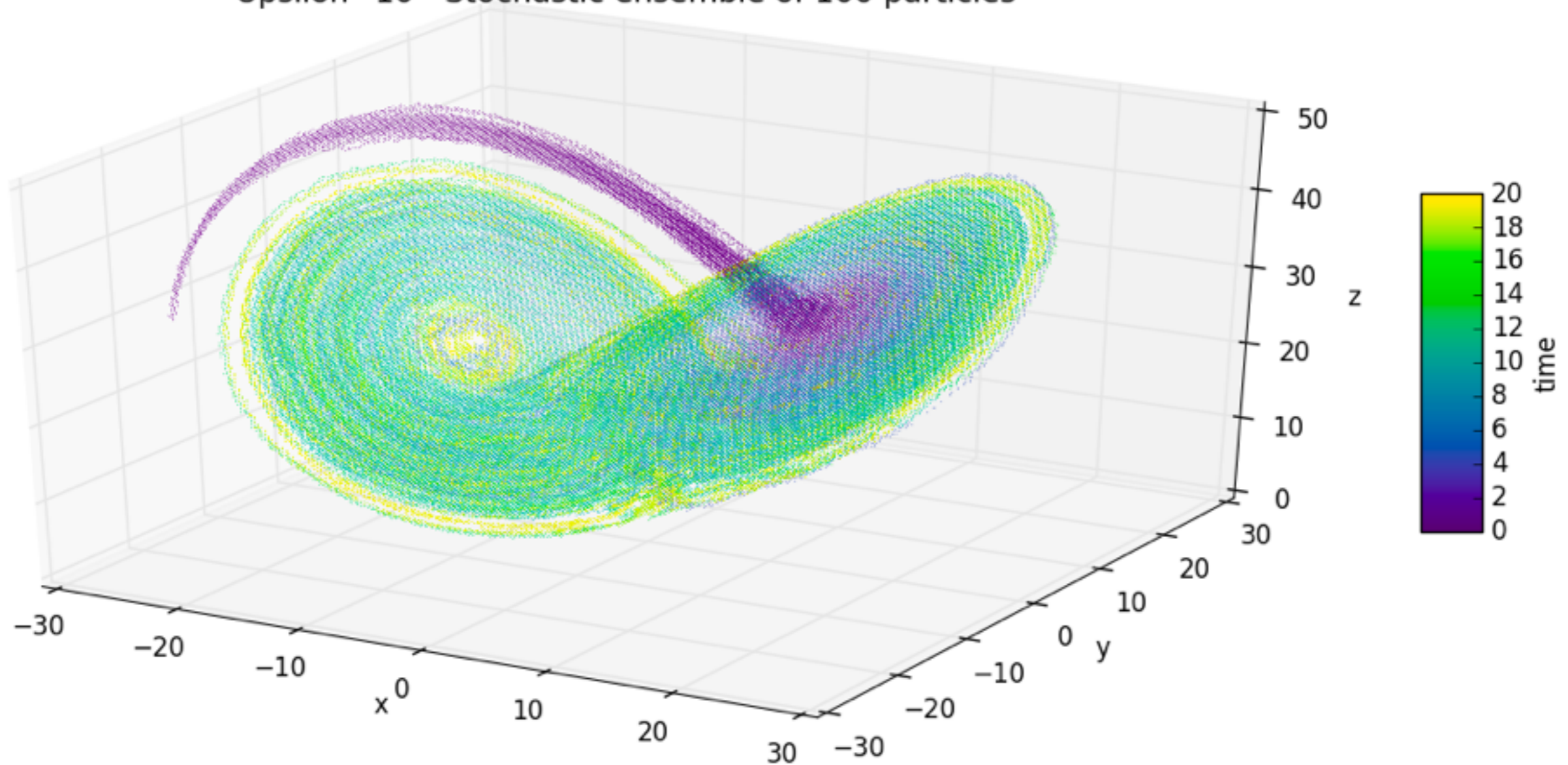
Upsilon=10 - Diffusive ensemble of 100 particles





Comportement à temps long

Upsilon=10 - Stochastic ensemble of 100 particles



Taux de visite de l'attracteur



Upsilon= 10 - attractor visited at T=0.00 by 100-particle ensembles



Taux de visite de l'attracteur



Upsilon= 10 - attractor visited at T=0.00 by 100-particle ensembles



Conclusion

Conclusion

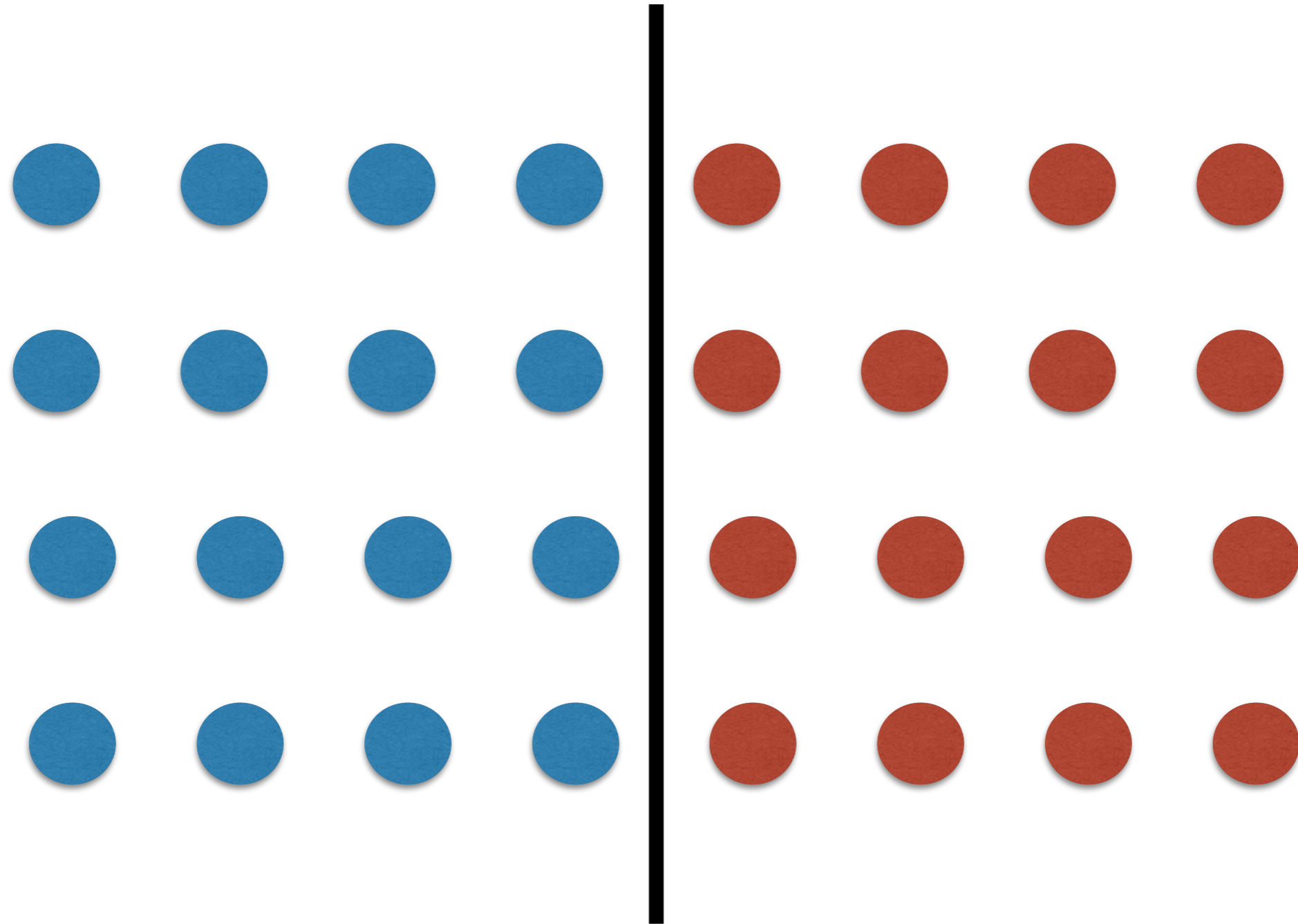
- Transport aléatoire applicable à n'importe quelle dynamique
- Petites échelles plus réalistes
- Dispersion efficace des ensembles
- Scénarios probables
- Exploration de l'attracteur

Merci de votre attention

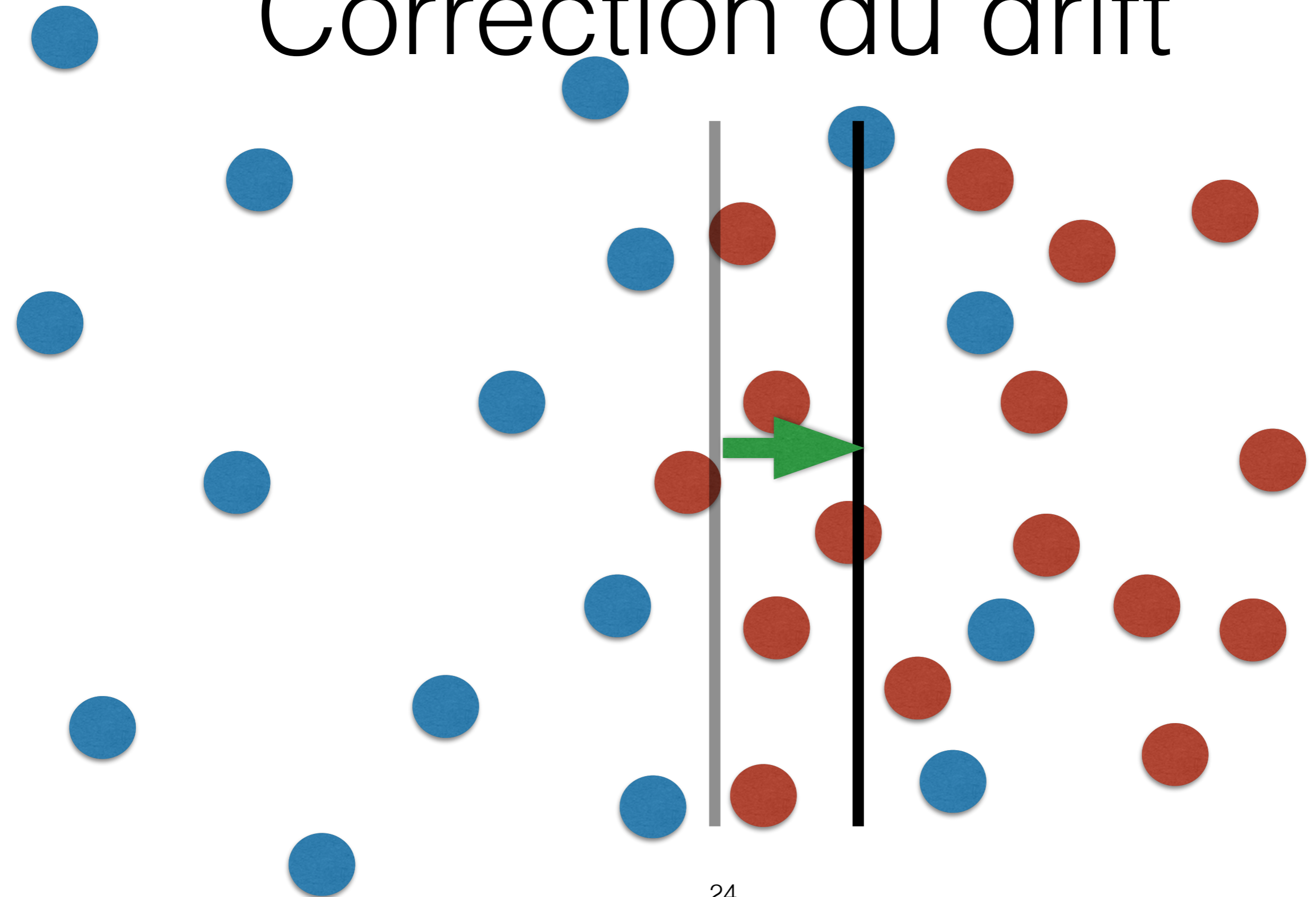
Code SQG MU:
lien depuis le site de l'équipe Fluminance - V. Resseguier

Correction du drift

Correction du drift

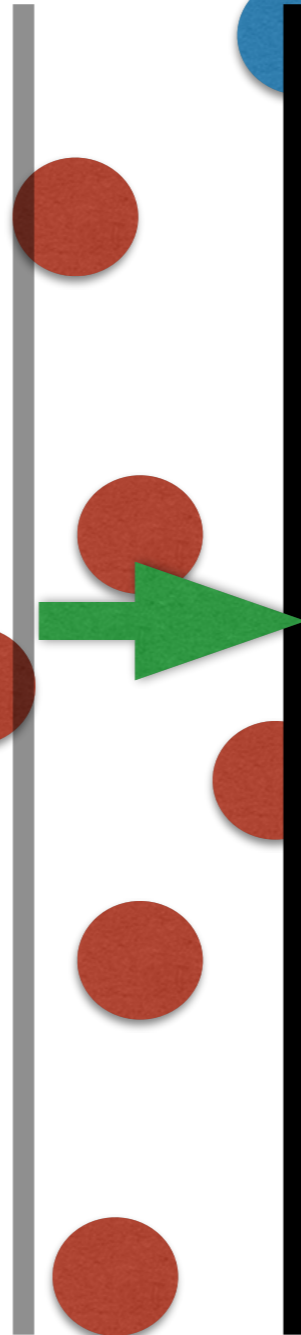


Correction du drift



Correction du drift

$$w^* = w - \frac{1}{2} (\nabla \cdot a)^T$$



Bifurcations dans une dynamique SQG

suivie avec SQG MU

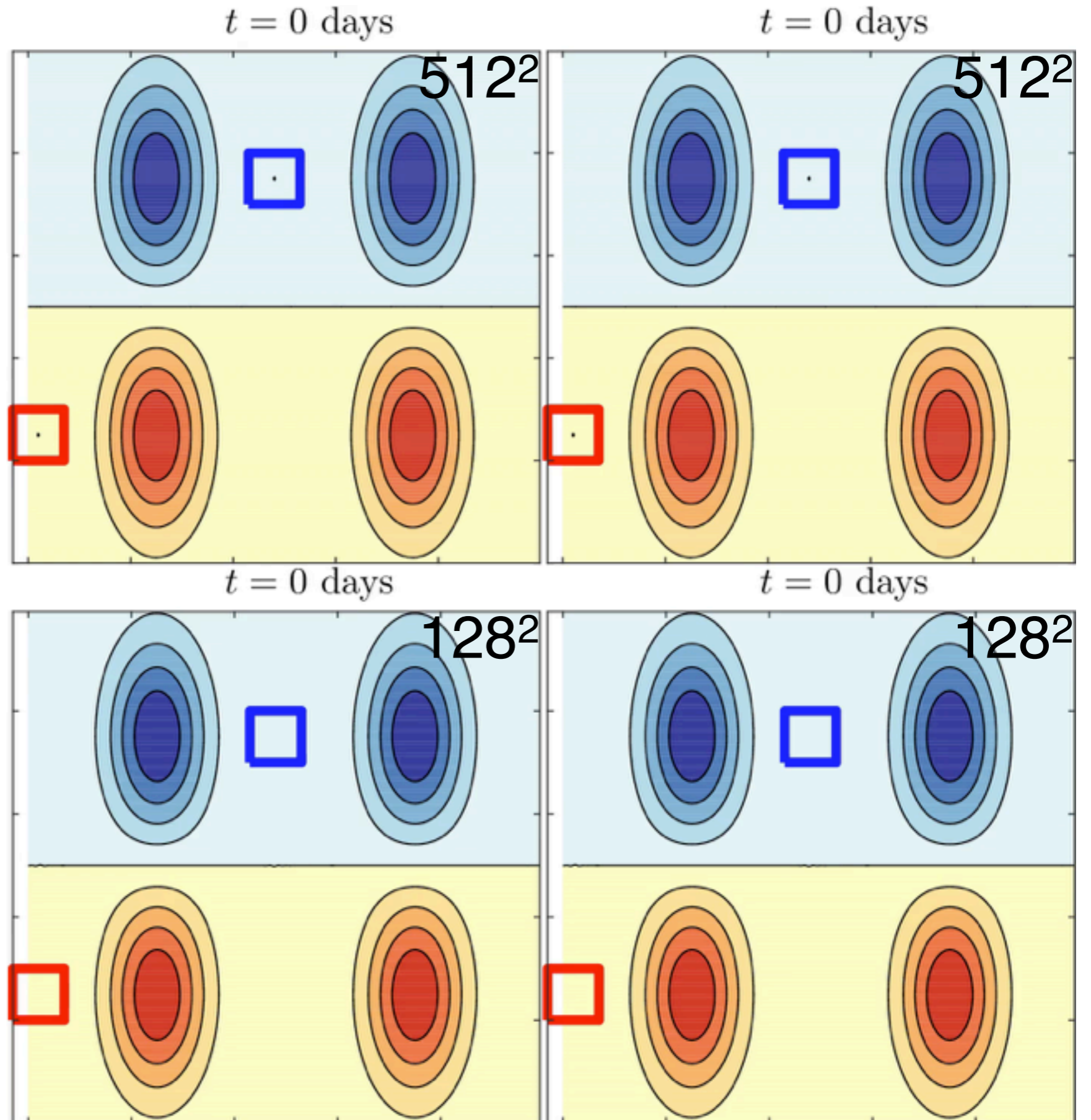
Simulation de
référence:
SQG déterministe

512² versus 128²

Condition initiale 1



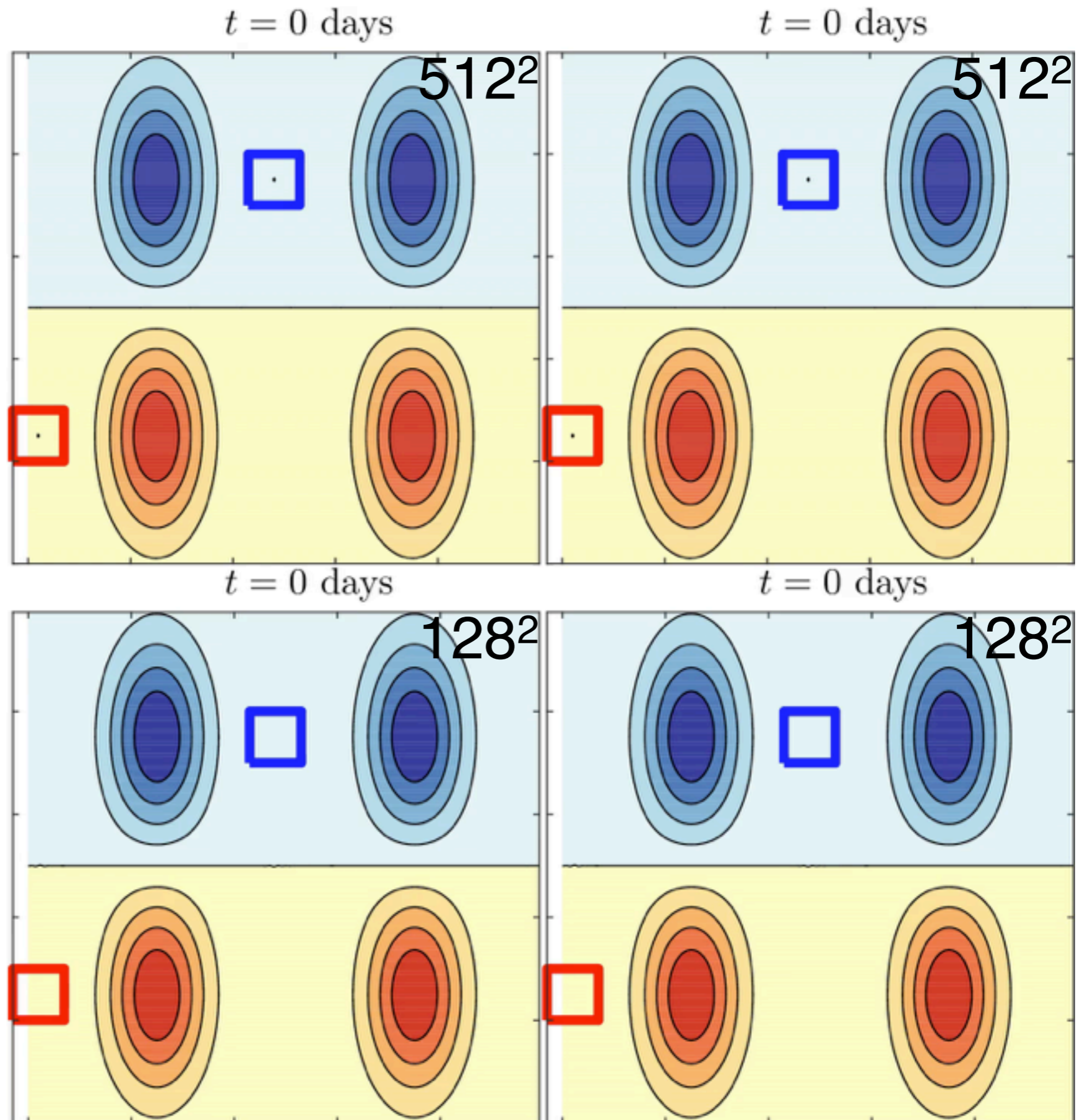
?

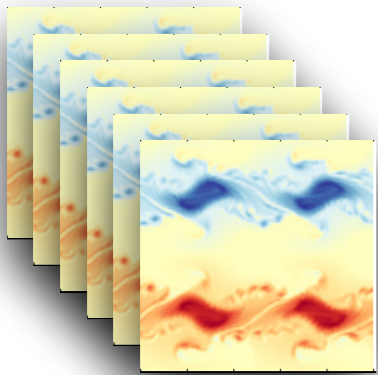


Simulation de
référence:
SQG déterministe

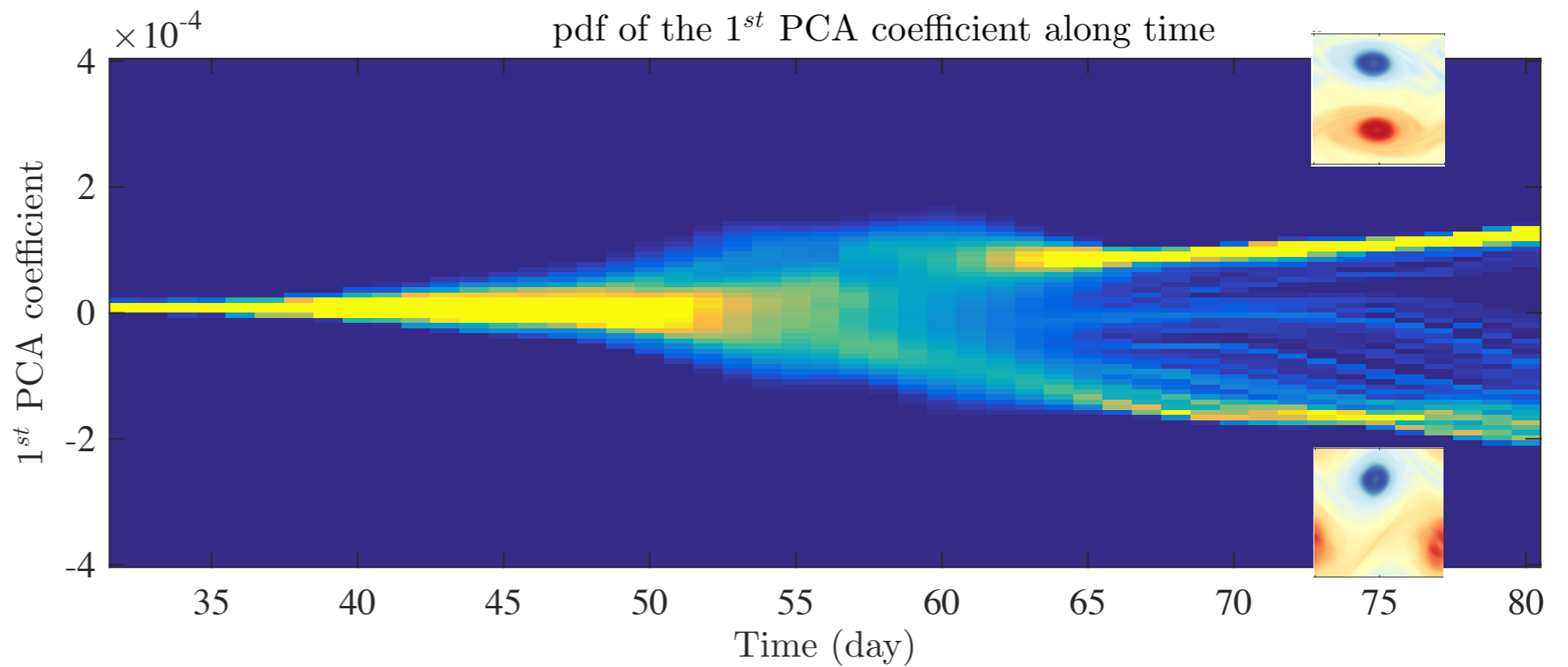
512² versus 128²

Condition initiale 1

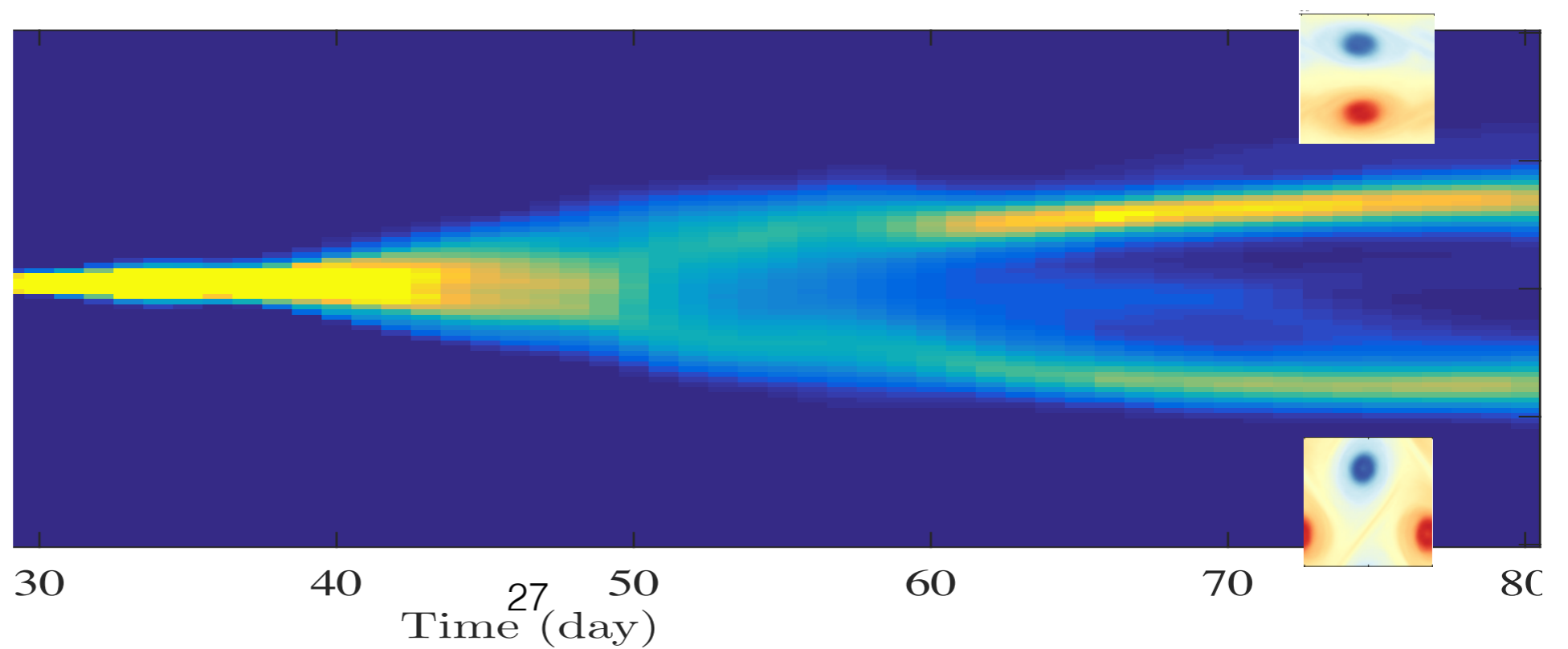




Conditions
initiales
aléatoires



Sous
incertitude
de position



SQG sous incertitude forte

SQG SU

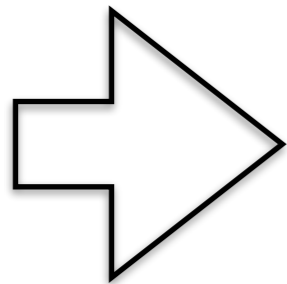
Divergence mésoéchelle

Equilibre géostrophique

$$\mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_b} \nabla p'$$

Diffusion
horizontale

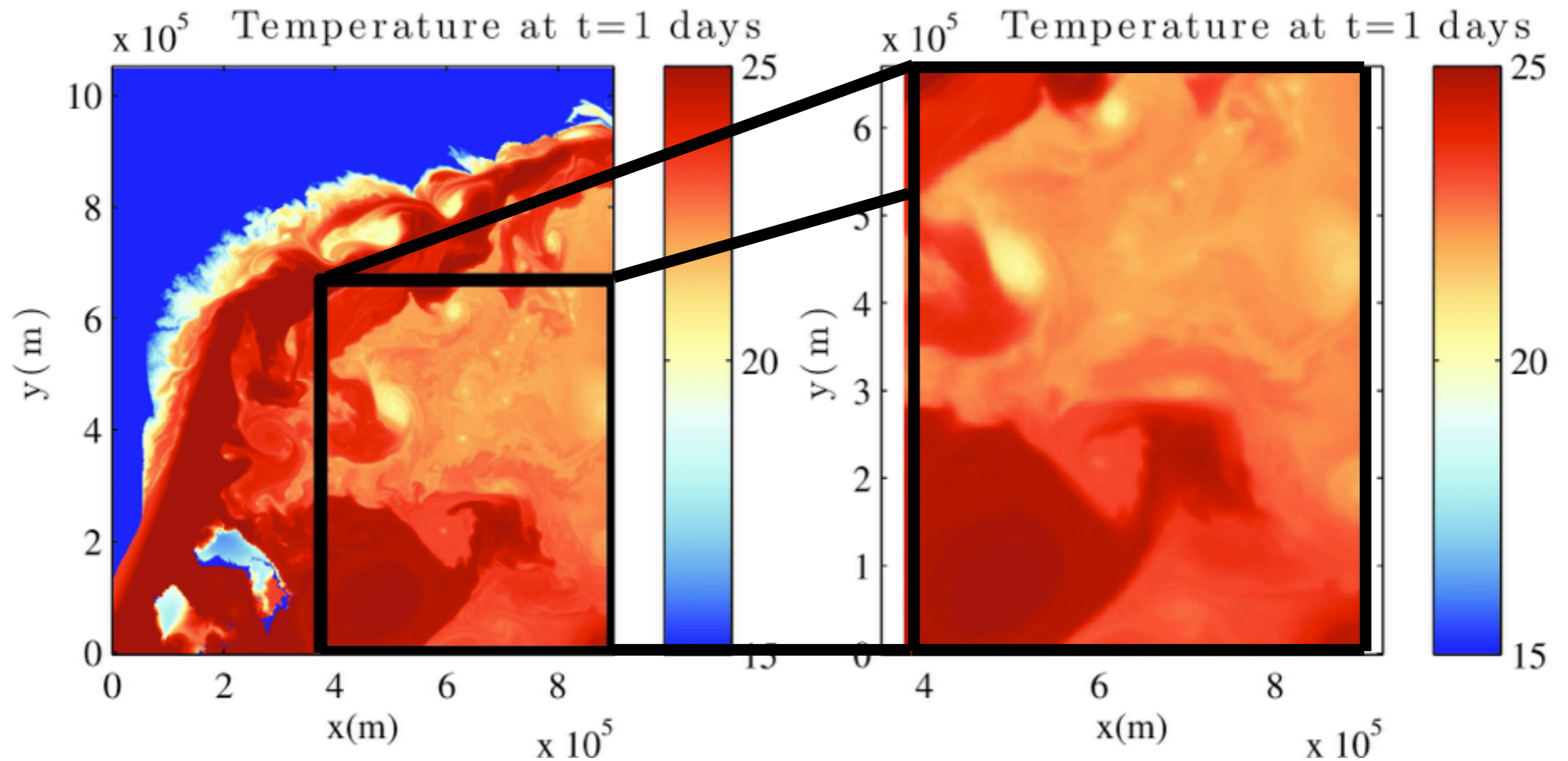
$$+ \frac{a}{2} \Delta \mathbf{u}$$



$$\nabla \cdot \mathbf{u} \propto \Delta \nabla^\perp \cdot \mathbf{u}$$

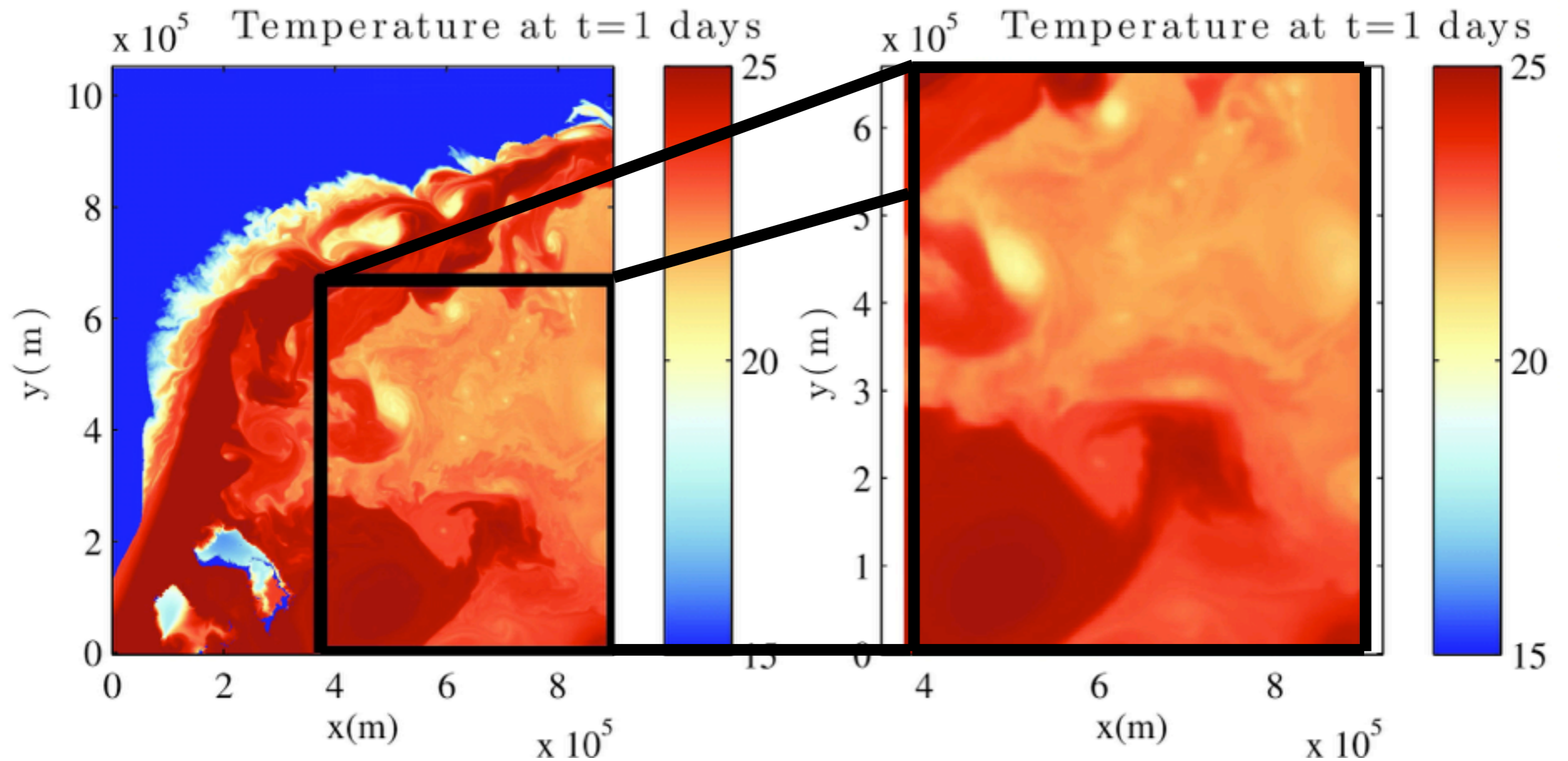
Filtrage des sorties de modèle:

Gula, Jonathan, M. Jeroen Molemaker, and James C. McWilliams
"Gulf Stream dynamics along the southeastern US seaboard."
Journal of Physical Oceanography 45.3 (2015): 690-715.

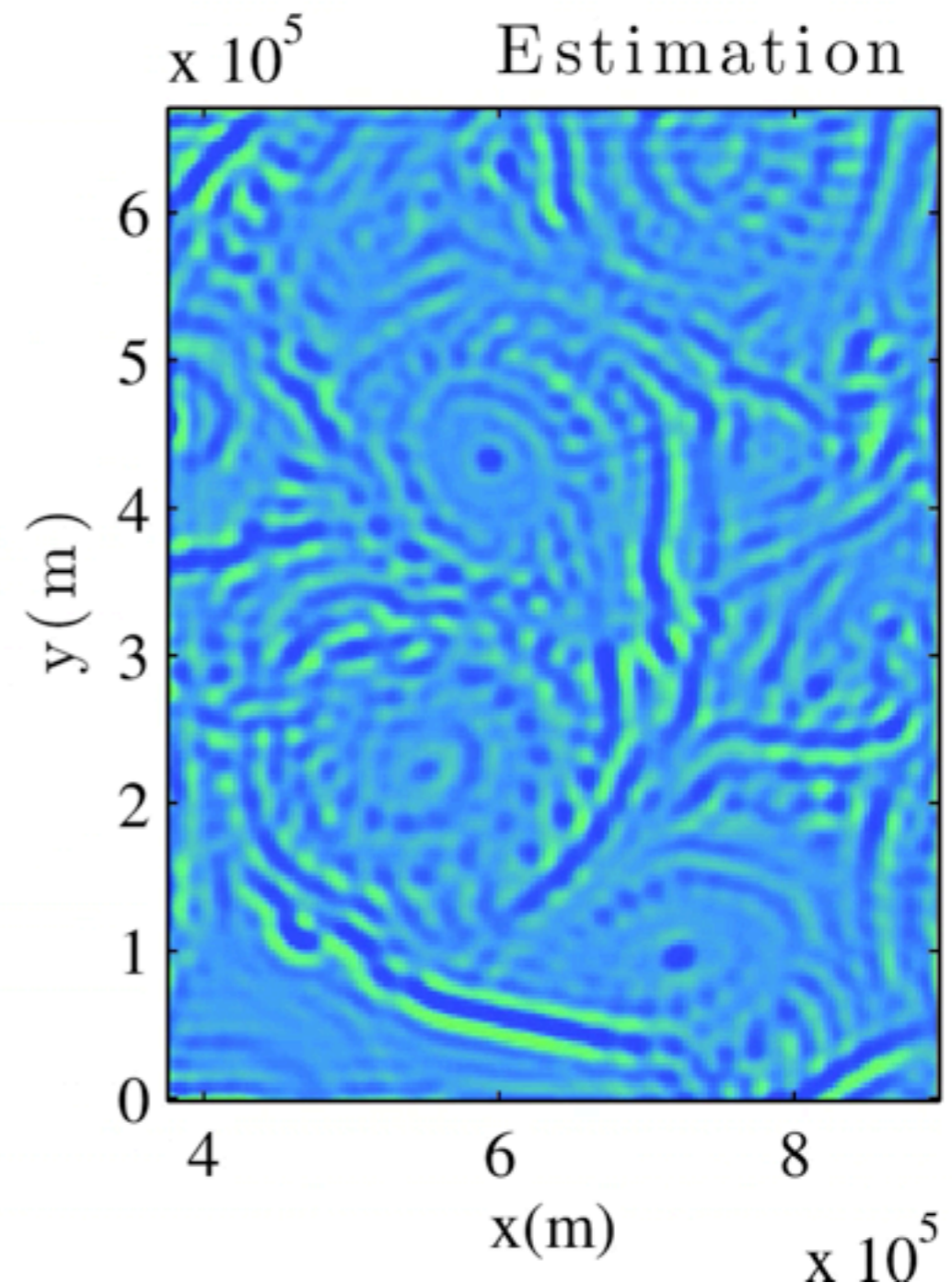
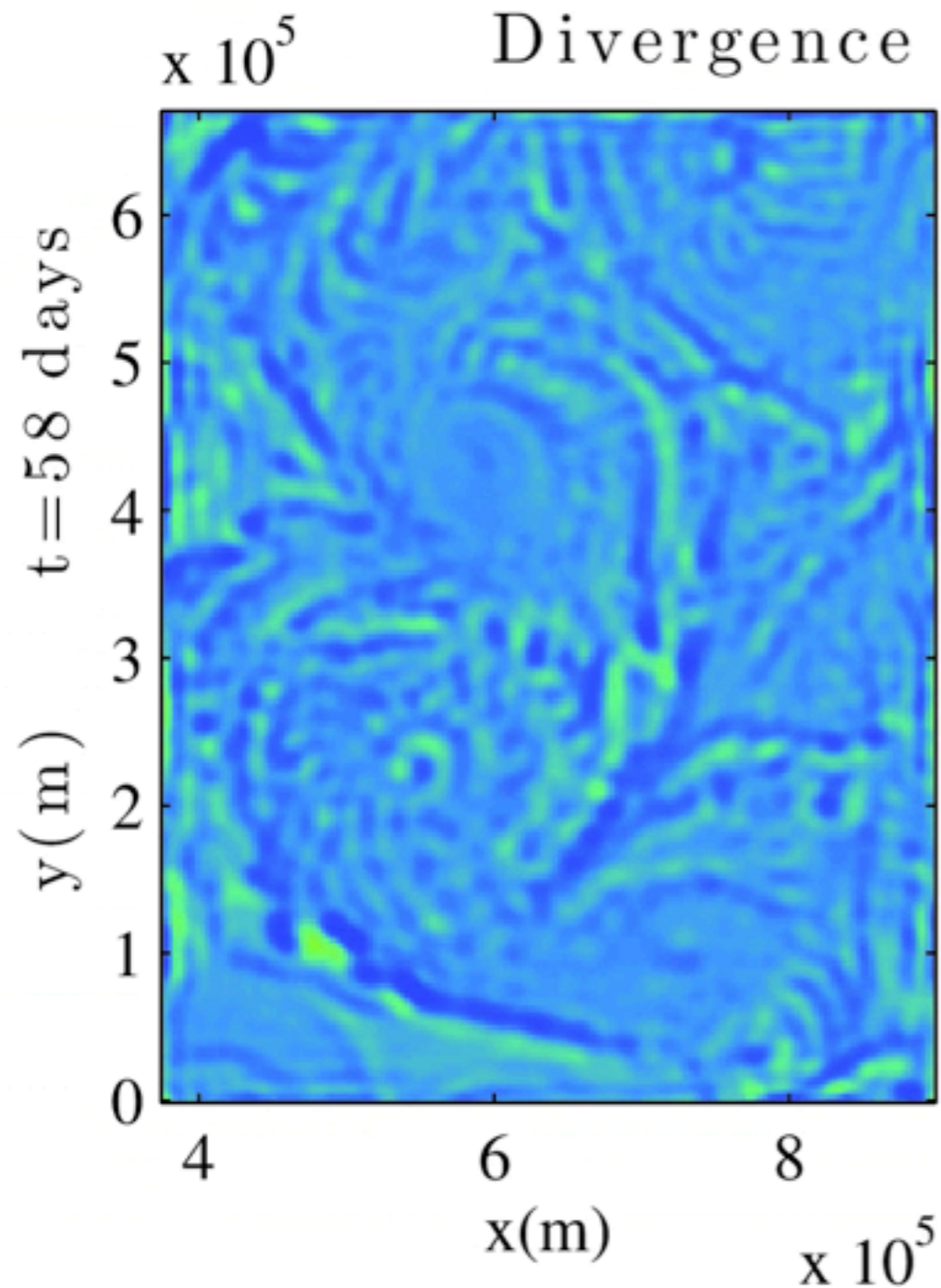


Filtrage des sorties de modèle:

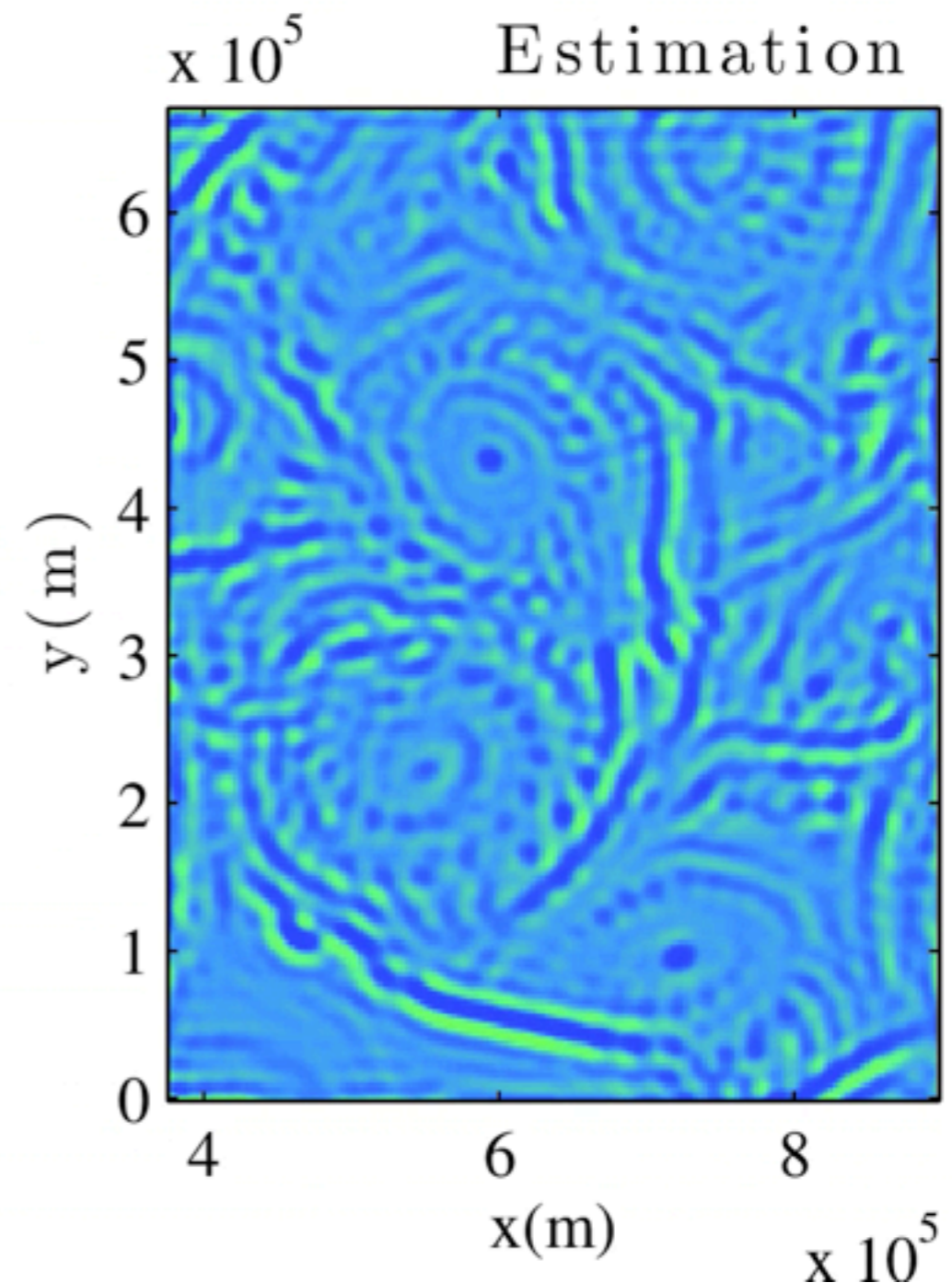
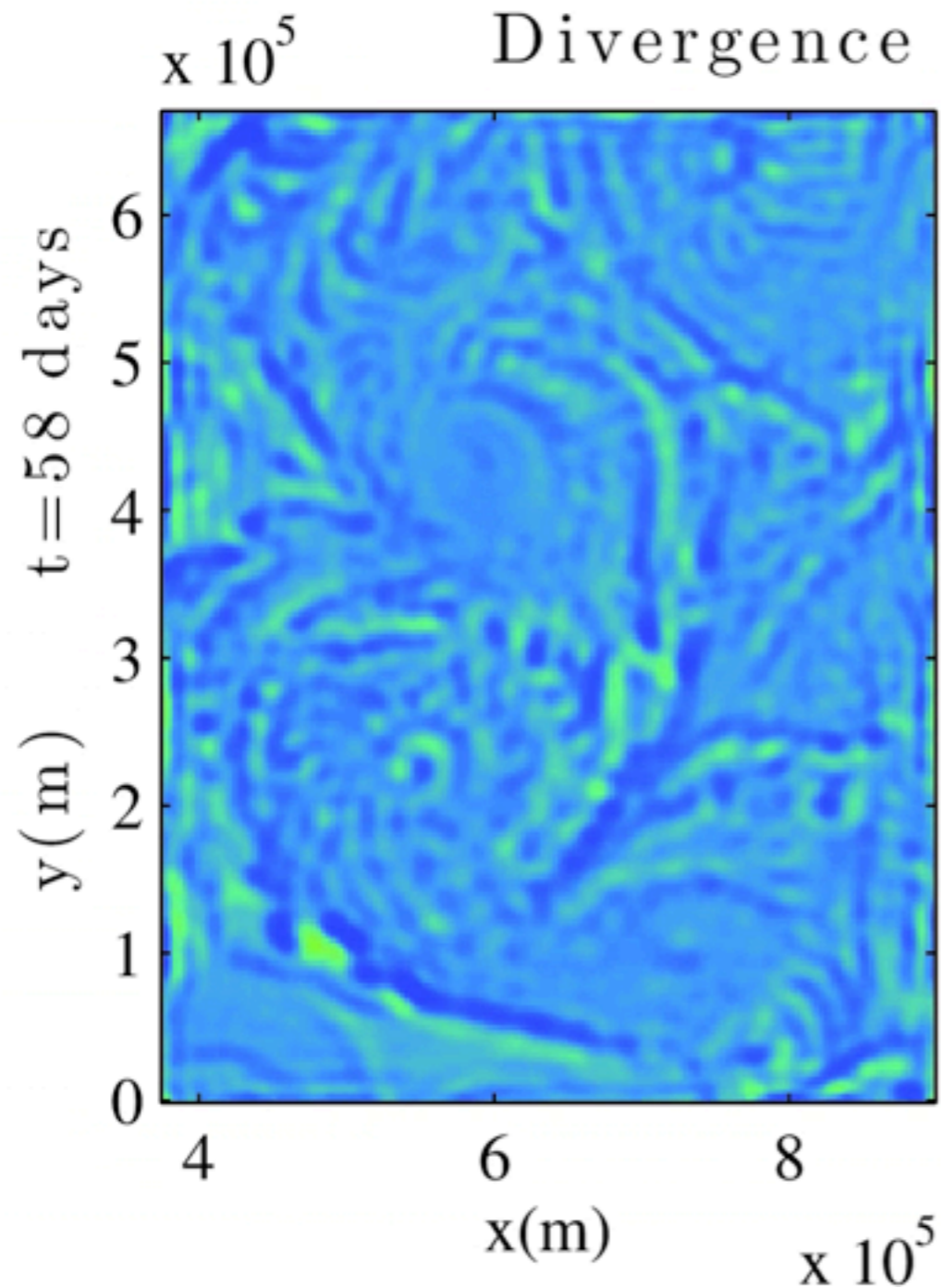
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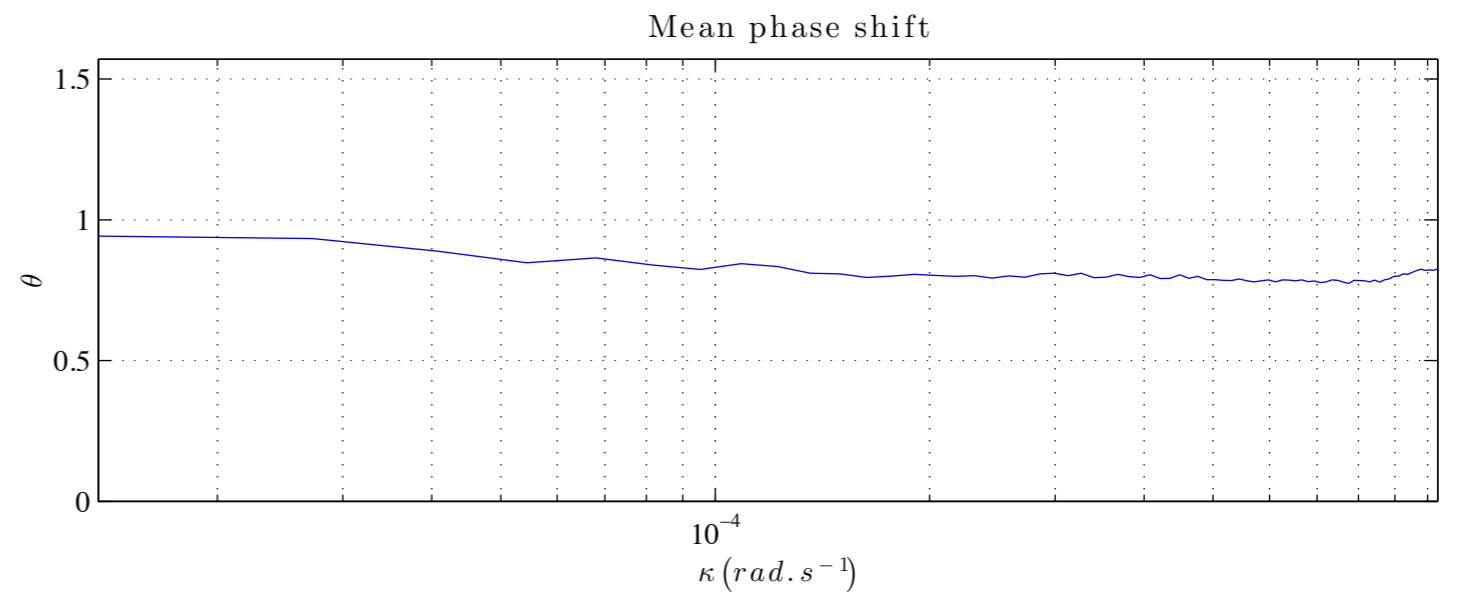
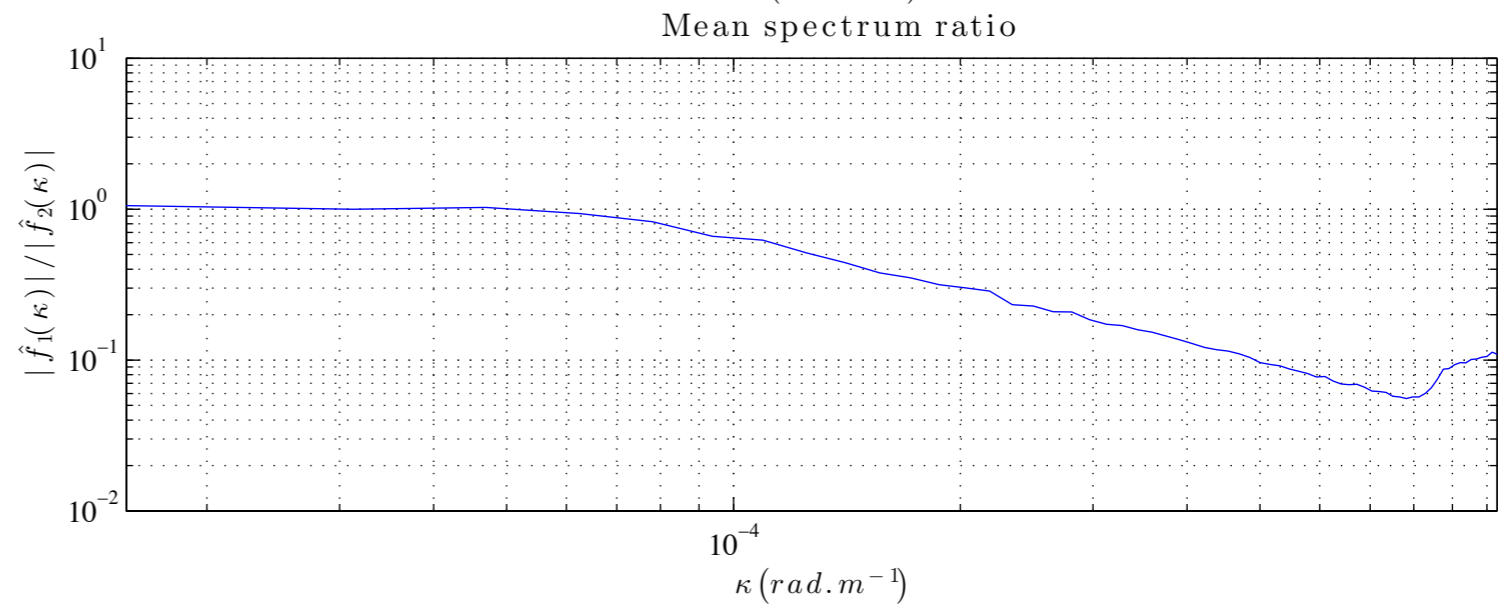
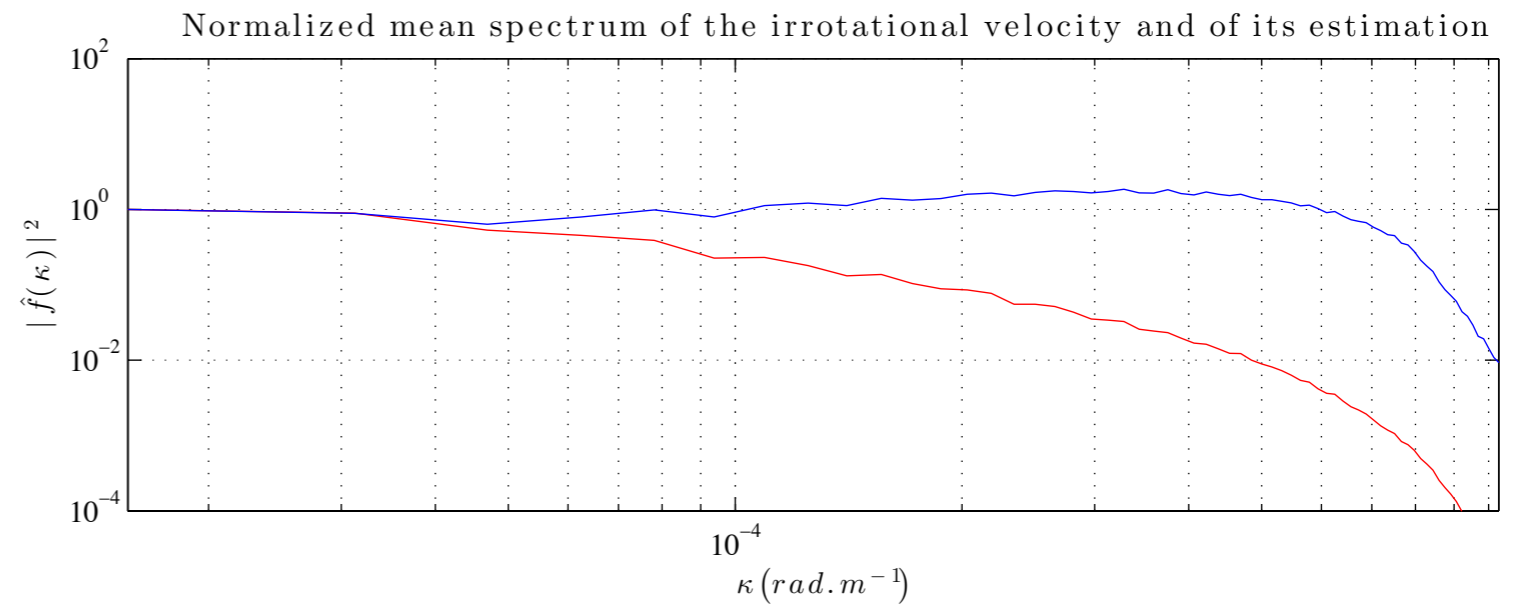
Test spatial



Test spatial



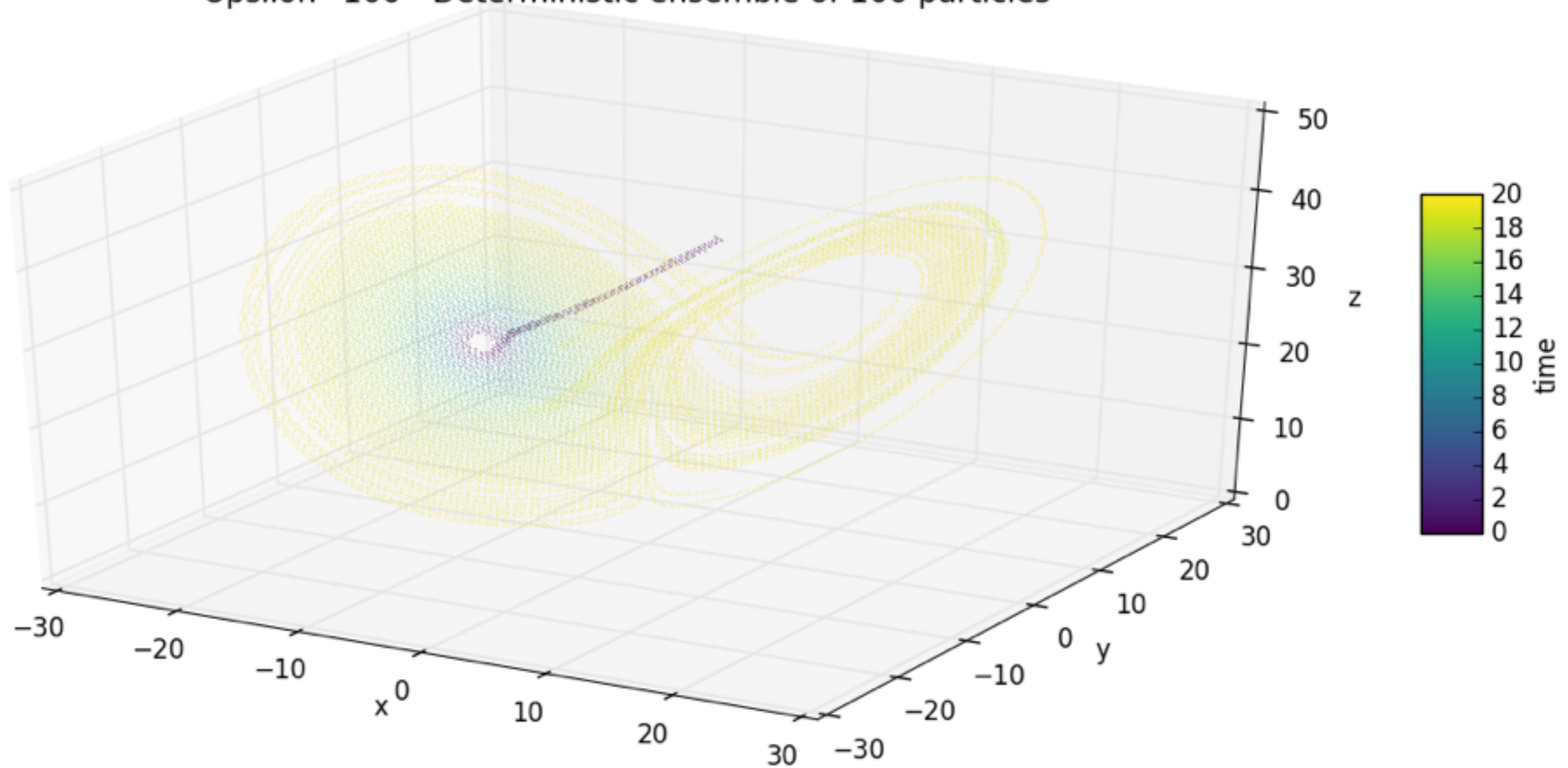
Test spectral





Temps long: Bruit faible

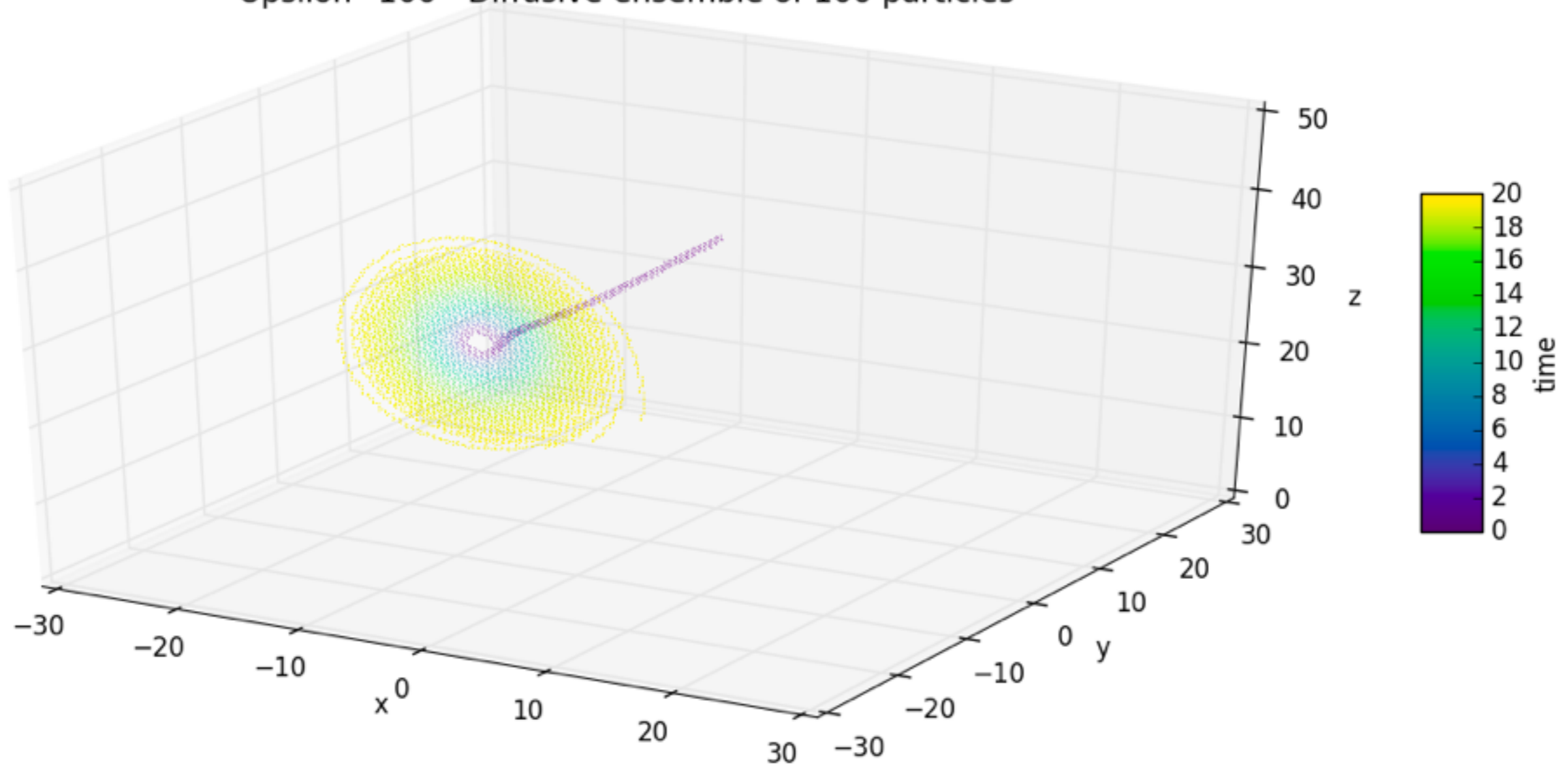
Upsilon=100 - Deterministic ensemble of 100 particles





Temps long: Bruit faible

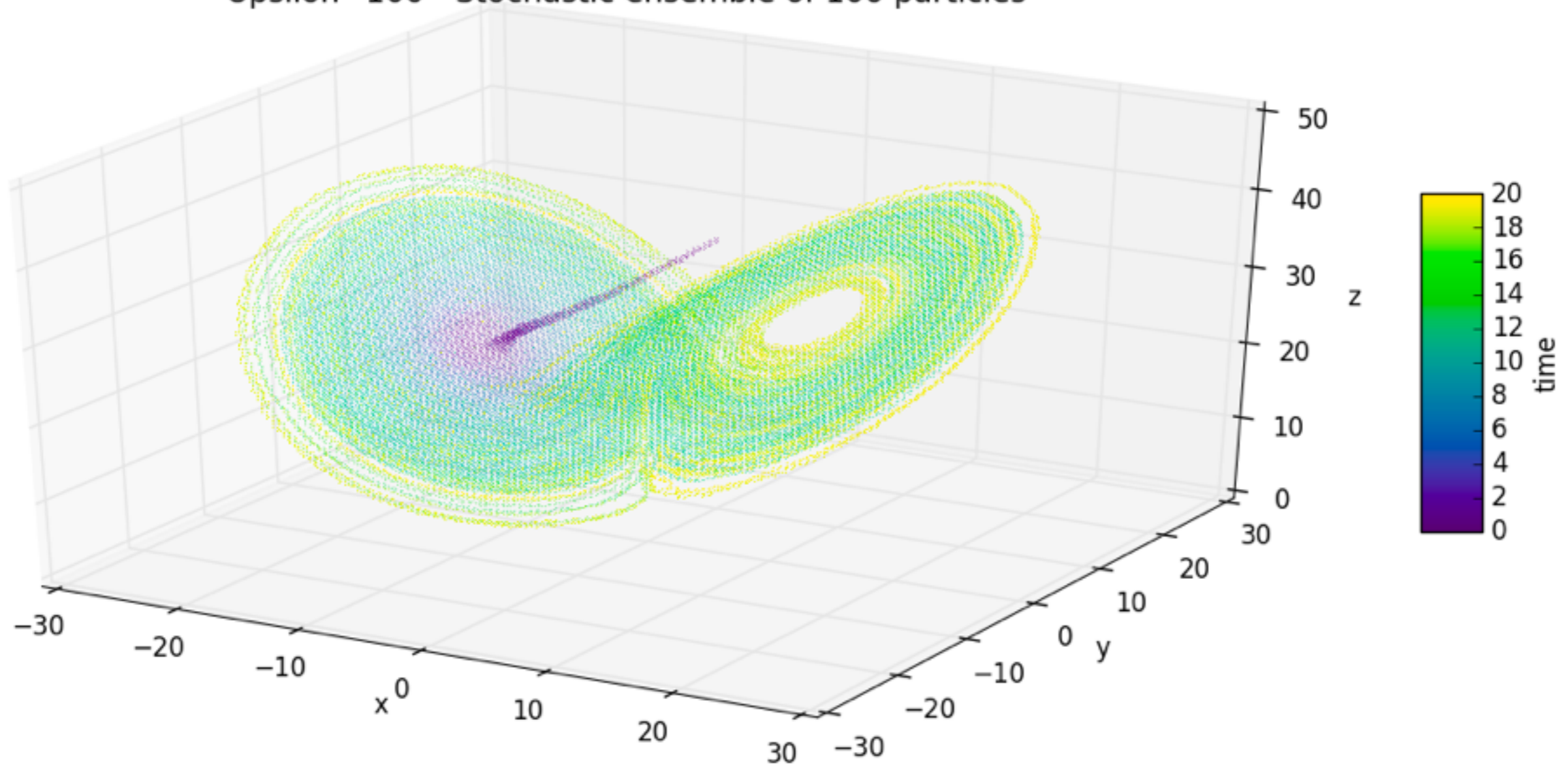
Upsilon=100 - Diffusive ensemble of 100 particles





Temps long: Bruit faible

Upsilon=100 - Stochastic ensemble of 100 particles



Taux de visite de l'attracteur



Upsilon=100 - attractor visited at T=0.00 by 100-particle ensembles



Taux de visite de l'attracteur



Upsilon=100 - attractor visited at T=0.00 by 100-particle ensembles





Calcul du taux de visite

- Recouvrement discret de l'attracteur de Lorenz (GAIO)
→ 611,550 boites cubiques de rayon=0.15625
- Pour chaque ensemble, le taux de visite:

$$\tau(T) = \frac{\#\{\text{unique boxes visited by ensemble over } [0; T]\}}{\text{total } \# \text{ of boxes}}$$

